

AD-A087 625

BOLT BERANEK AND NEWMAN INC CAMBRIDGE MA
THE STRUCTURE OF ANALOGICAL MODELS IN SCIENCE.(U)
JUL 80 D GENTNER

F/6 5/10

N00014-79-C-0338

UNCLASSIFIED

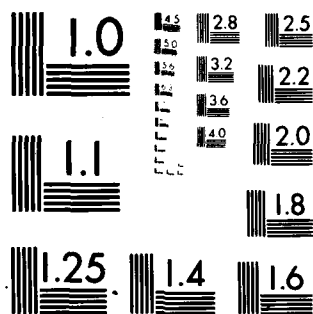
BBN-4451

ML

For
9/1/80



END
DATE
FILMED



MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

54
Bolt Beranek and Newman Inc.

(12)
bbn

LEVEL 4

Report No. 4451

ADA 087625

The Structure of Analogical Models in Science

Dedre Gentner

July 1980

DISTRIBUTION STATEMENT A

**Approved for public release;
Distribution Unlimited**

**Prepared for:
Office of Naval Research and the
Defense Advanced Research Projects Agency**

**DTIC
ELECTE**

AUG 7 1980

A

**This research was sponsored by the Personnel and Training Sciences
Division, Office of Naval Research, under Contract No. N00014-79-C-0338,
Contract Authority Identification No. NR157-428. Approved for public release;
distribution unlimited. Reproduction in whole or in part is permitted for any
purpose of the United States Government.**

DC FILE COPY

80 8 7 07

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(14) BBN-4451, TR-2

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Technical Report No. 2	2. GOVT ACCESSION NO. AD A087625	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) The Structure of Analogical Models in Science.	5. TYPE OF REPORT & PERIOD COVERED Semiannual Technical Report September 1979-March 1980	6. PERFORMING ORG. REPORT NUMBER BBN Report No. 4451
7. AUTHOR(s) Dedre Gentner	8. CONTRACT OR GRANT NUMBER(s) N00014-79-C-0338	9. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
10. PERFORMING ORGANIZATION NAME AND ADDRESS Bolt Beranek and Newman Inc. 50 Moulton Street Cambridge, Massachusetts 02138	11. CONTROLLING OFFICE NAME AND ADDRESS Personnel and Training Research Programs Office of Naval Research (Code 458) Arlington, Virginia 22217	12. REPORT DATE Jul 1980
13. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (12) 95	14. SECURITY CLASS. (of this report) UNCLASSIFIED	15. NUMBER OF PAGES 79
16. DISTRIBUTION STATEMENT (of this Report) APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED		17. SECURITY CLASS. (of this report) UNCLASSIFIED
18. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
19. SUPPLEMENTARY NOTES To be submitted to <u>Cognitive Science</u>		
20. KEY WORDS (Continue on reverse side if necessary and identify by block number) Metaphor, Analogy, Structure Mapping		
21. ABSTRACT (Continue on reverse side if necessary and identify by block number) Analogical models can be powerful aids to reasoning, as when light is explained in terms of water waves; or they can be misleading, as when chemical processes are thought of in terms of life processes such as putrefaction. This paper proposes a structural characterization of good science analogy using a theoretical approach in which complex metaphors and analogies are treated as <u>structure-mappings</u> between domains. To delineate good from poor science analogy, a series of comparisons is made. First, metaphor and		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 68 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

060100

Jm

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. analogy are contrasted with literal similarity; then, explanatory-predictive analogy is contrasted with expressive metaphor; finally, within science, good explanatory analogy is contrasted with poor explanatory analogy. Analogies of historical importance are analyzed and empirical findings are discussed.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

A

A

1990

(continued from page 70)

A

A

A

A

Abstract

Analogical models can be powerful aids to reasoning, as when light is explained in terms of water waves; or they can be misleading, as when chemical processes are thought of in terms of life processes such as putrefaction. This paper proposes a structural characterization of good science analogy using a theoretical approach in which complex metaphors and analogies are treated as structure-mappings between domains. To delineate good from poor science analogy, a series of comparisons is made. First, metaphor and analogy are contrasted with literal similarity; then, explanatory-predictive analogy is contrasted with expressive metaphor; finally, within science, good explanatory analogy is contrasted with poor explanatory analogy. Analogies of historical importance are analyzed and empirical findings are discussed.

And I cherish more than anything else the Analogies,
my most trustworthy masters. They know all the
secrets of Nature, and they ought to be least
neglected in Geometry.

-Kepler (quoted in Polya,
1973)

Isaac Newton likened the moon to a ball thrown so hard
that its downward fall misses the earth and it passes into
orbit. Galileo compared the moon, were it to fall out of its
orbit, to a rock dropped from the mast of a moving ship: its
motion would have both a falling component and a forward
component shared with the ship. Both these analogies made the
moon's motion appear a combination of falling and moving
forward. It became clear that, to preserve a circular path,
in each instant the moon's tangential displacement must
compensate for its inward displacement. These analogies
played a role in the shift away from the deeply held
Aristotelian view that a body could have only a single motion,
and that circular motion was an essential quality of heavenly
bodies, to the view that the orbits of the moon and planets
are composite motions.

Models that explain a new topic by analogy with a
familiar domain are common in science. Other examples are
Rutherford's comparison of the atom to the solar system; the

analogy between propagation of sound in air and propagation of waves in water, and the further analogy to propagation of light through space; and the hydraulic model of electric circuitry. Current work in nuclear physics likens weak interactions among elementary particles a field induced by a weak, uncharged electric current. Finally, a familiar but useful example is the standard mathematical technique of analogizing from two- or three-dimensional spaces to n-dimensional spaces.

Yet metaphorical thinking can foster vagueness. In alchemical analogy, chemical processes were explained in terms of correspondences with life processes and psycho-spiritual processes such as debasement and redemption. For example, in the putrefaction stage of a chemical reaction, a black, foul-smelling chemical was supposed to give rise to a more vital material, just as rotting mud was believed to engender life. This set of correspondences persisted for a very long time, and may have impeded progress in chemistry (Cavendish, 1967). There are examples closer to hand of analogies whose usefulness is debatable. There is the "urban blight" metaphor by which terms like "afflicted" and "organically sound" are applied to neighborhoods (Lakoff & Johnson, in press; Schon, 1979). Psychology has used terms such as "reverberating circuits", "mental distance", "perceptual defense", "memory

capacity", "mental image" and "depth of processing" that have at least a partly metaphorical status. Some of these analogies have suggested deep research, while others have merely provided a kind of spurious feeling of comfort (see Pylyshyn, 1979).

What makes some analogies useful in scientific thinking and others useless or harmful? One might propose that good analogies are those that make correct predictions while bad analogies make false predictions. This proposal is inadequate. As we will see, good analogies make incorrect as well as correct predictions; and even primarily incorrect analogies can lead to useful research. Yet there are, I believe, definable differences between good and poor explanatory analogies.

The goal of this paper is to provide a structural characterization of good science analogy. The plan is, first, to present a theoretical approach in which complex metaphors and analogies are treated as structure-mappings between domains. Within this framework, a series of increasingly closer comparisons is used to help delineate structural characteristics of good scientific analogy. First, metaphor and analogy are contrasted with literal similarity; then, explanatory-predictive analogy is contrasted with literary expressive metaphor. Finally, within science, good

explanatory analogy is contrasted with poor explanatory analogy. The characterization of scientific analogies leads to consideration of the processes of development of a science model from an initial comparison. To illustrate the points, analogies of historical importance are analyzed. Although the focus is on theory, some empirical findings will also be discussed.

Models as Structure Mappings

The first point is a terminological one. There is no good term for "nonliteral similarity comparison." The term "metaphor" conveys an artistic or expressive nonliteral comparison of a certain form; the term "model" conveys an explanatory-predictive nonliteral comparison, often mathematically stated. Since I want to discuss the structure of both metaphors and models, I need a neutral term. I will use the term "analogy" as a general term for nonliteral similarity comparisons, including metaphors, similes, and models. In cases when the narrow sense of "analogy" as a comparison of the form $A:B::C:D$ is needed, I will use the term "simple analogy".

The models used in science belong to a large class of analogies that can be characterized as structure-mappings between complex systems. Typically, the target system to be understood is new or abstract, and the base system in terms of

which the target is described is familiar and perhaps visualizable. In these analogies, the objects of the known domain are mapped onto the objects of the domain of inquiry, allowing the predicates of the first domain to be applied in the other domain. Further, among the base predicates, it is primarily the relations that hold among the nodes of the base domain that are applied to the nodes of the target domain. Thus, a structure-mapping analogy asserts that identical operations and relationships hold among nonidentical things. The relational structure is preserved, but not the objects. For example, Polya (1973) states " . . . [in] the most typical case of clarified analogy, . . . two systems are analogous, if they agree in clearly definable relations of their parts." The notion of system is important here. To perform such a structure-mapping requires viewing both domains as systems of objects and relationships. (The "objects" need not be concrete separable objects, but they must at least be separable components.)

Given the importance of scientific analogies it is perhaps surprising that they have received so little attention in psychology. The major reason, I suspect, is that these analogies are comparisons between systems and cannot be analyzed as single object comparisons. Most psychological treatments of metaphor are aimed at object-object comparisons,

such as ". . . my love is like a red red rose. . . ." These treatments are based either on feature-list descriptions or multidimensional space descriptions of the domains, both of which can deal with object attributes but not with relations between objects. A different approach is required to characterize analogies between complex systems of interrelated objects. (A fuller discussion of other approaches is given in the latter part of this paper.)

The structure-mapping approach is componential, in that two different domains are assumed to share some parts of their knowledge representations. In this respect it resembles a feature-list approach. However, unlike a feature-list theory, the structure-mapping approach makes a strong distinction between objects and their attributes, on the one hand, and relationships, on the other hand. This approach thus requires fairly well-elaborated representations of meaning in both domains, such as are provided by a schema-theoretic representation. By "schema" is meant a knowledge structure expressed in terms of a set of concepts and predicates over those concepts (Bobrow, 1975; Rumelhart & Norman, 1975; Rumelhart & Ortony, 1977; Schank & Abelson, 1977). For present purposes, the most useful representation of a schema is as a propositional network of nodes and predicates. The nodes represent concepts treated as wholes and the predicates

express propositions about the nodes. These schemas are hierarchical: a node at one level may decompose at a lower level into another network of nodes and relationships. However, at any given level of representation a topic area can be characterized in terms of nodes and predicates, where the predicates can be either attributes - predicates taking one argument - or relations - predicates taking two or more arguments. For example, COLLIDE (x,y) is a relation, while RED (x) is an attribute.

Given such a propositional representation, we can proceed with the characterization of a metaphor or analogy as a structure-mapping between a known domain (the base domain) and a domain of inquiry (the target domain). (cf. Brown, Collins & Harris, 1978; Gentner, 1977 a; Gentner, 1977 b; Miller, 1979; Rumelhart, 1979.) A structure-mapping analogy between a target system T and a base system B is an assertion that

- (1) there exists a mapping M of the nodes b_1, b_2, \dots, b_n of system B into the (different) nodes t_1, t_2, \dots, t_m of system T.
- (2) The mapping is such that substantial parts of the relational-operational structure of B apply in T: that is, many of the relational predicates that are valid in B must also be valid in T, given the node substitutions dictated by M:

$\text{TRUE } [F(b_i, b_j)] \text{ implies } \text{TRUE } [F(t_i, t_j)]$.

Assertions (1) and (2) define the basic structure-mapping. However, they are also compatible with a general similarity relationship between the domains T and B. To specify that the match is one of analogical relatedness and not literal similarity, we need a further stipulation:

- (3) Relatively few of the valid attributes (the one-place predicates) within B apply validly in T.

$\text{TRUE } [A(b_i)] \text{ does not imply } \text{TRUE } [A(t_i)]$.

Assertions (2) and (3), taken together, state that relational predicates, and not object attributes, carry over in analogical mappings. This follows from the central assertion that such mappings apply the same relations to different objects.

Analogy versus similarity. The degree of matching among objects versus relations determines whether a comparison statement will convey literal similarity or analogical relatedness. When both the component objects and the relational structure overlap, the comparison is one of literal similarity. An example is

- (1) The helium atom is like the neon atom.

This is a literal similarity comparison, because there is considerable overlap both in the component objects - protons,

neutrons and electrons - and in the relations between those objects - e.g., "electron REVOLVES AROUND (proton AND neutron)". Note that not all the objects and relations correspond perfectly; if they did, the statement would convey identity, not similarity.²

If the relationships correspond, but the objects do not, the comparison is analogical. An example is

(2) The hydrogen atom is like the solar system.

Here, the component objects are totally different; what the statement conveys is overlap in the relational structures of the two systems.

The final possibility is to have overlap among objects but not among relationships. This represents neither literal nor analogical similarity. Such comparisons are rarely of interest. Perhaps the clearest instances are chemical equations, in which atoms (the objects) are rearranged from one molecular grouping (set of structural relations) to another; for example,

(3) $\text{CaCO}_3 = \text{CaO} + \text{CO}_2$.

This equation conveys that the molecules of calcium, carbon and oxygen that make up limestone (calcium carbonate, CaCO_3) can be rearranged to form lime (CaO) and carbon dioxide (CO_2). Here the two sides of the equation are neither literally nor analogically similar. We do not say that limestone is like

lime and carbon dioxide. Their connection seems rather one of chronological relatedness: the two configurations can apply to the same objects (atoms) at different times.

To summarize, overlap in relations is necessary for the perception of similarity between two domains. Overlap in both object attributes and inter-object relationships is seen as literal similarity; overlap in relationships but not objects is seen as analogical relatedness; and overlap in objects but not relationships is seen as temporal relatedness, not as similarity. According to this brief demonstration, no featural treatment of analogical or metaphorical similarity can be complete without distinguishing between object features and relational features: that is, between relational predicates and one-place attributive predicates. A further implication is that literal similarity versus metaphorical relatedness is a continuum, not a dichotomy. Given that two domains overlap in relationships, they are more literally similar to the extent that their component object-attributes also overlap.

The assumption that predicates are brought across as identical matches is crucial to the clarity of this discussion. The position that predicates need only be similar between the base and the domain (e.g., Hesse, 1966; Ortony, 1979) leads to a problem of infinite regress, with similarity

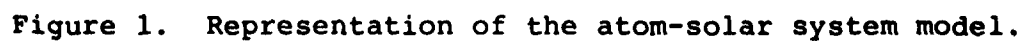
of surface concepts defined in terms of similarity of components, etc,. The assumption I make instead is that similarity can be restated as partial identity: that predicates can be decomposed into lower-level predicates, and that a high-level similarity match can be reformulated as an identity match among some number of the component predicates.

Let us briefly examine a simple analogy before proceeding to a more complex case. A simple arithmetic analogy, such as $3:6::2:4$, is the simplest case of structure-mapping. If we make the mapping of 2 onto 3 and 4 onto 6, we find that the relation "denominator TWICE AS LARGE AS numerator" holds between 3 and 6 as it does between 2 and 4. Dissimilar objects exist in the same relationships. This a particularly simple case, first because the relevant relation (proportionality) is understood by convention, and second, because only one relationship is involved. In complex analogies, it can be harder to identify the relations that are to be mapped; and there may be several different mapped relationships.

The atom/solar system analogy. An example of a complex analogy is Rutherford's solar system model of the hydrogen atom. Figure 1 shows the structure-mapping conveyed by this analogy. Starting with the known base domain of the solar system, the object-nodes of the base domain (the sun and

planets) are mapped onto object-nodes (the nucleus and electrons) of the atom. Given this correspondence of nodes, the analogy conveys that the relationships that hold between the nodes in the solar system also hold between the nodes of the atom: for example, that there is a force attracting the peripheral objects to the central object; that the peripheral objects revolve around the central object; that the central object is more massive than the peripheral objects; and so on.

This example shows how objects and their attributes are treated differently from relations in the mapping process. Base objects are mapped onto quite dissimilar target objects (e.g., the sun onto the nucleus). It is the relations in the base domain that are preserved. For example, the ATTRACTS relation and the REVOLVES AROUND relation between planet and sun are carried across to apply between electron and nucleus, while the separable attributes of the base objects, such as the color or temperature of the sun, are left behind.³ Mass provides a good illustration: The relation "MORE MASSIVE THAN" between sun's mass and planet's mass carries over, but not the absolute mass of the sun. We do not expect the nucleus to have a mass of 10^{30} kilograms, any more than we expect it to have a temperature of $25,000,000^{\circ}\text{F}$. The analogy conveys that the two domains, though composed of different objects, share much of their relational structure.



Galileo's earth/ship analogy. Another example of complex modelling is an analogy that Galileo uses in a dialogue concerning whether the earth rotates (Galileo, 1638; translated by Drake, 1967; p. 144). The new Copernican view that the earth does rotate is argued by Salviati, Galileo's surrogate; the prevailing Aristotelian view that the earth is the unmoving center of the universe is argued by the philosopher Simplicius. They are discussing a proof that the earth stands still: the fact that a stone dropped from a high tower drops straight down, instead of falling behind the tower, as it would if the earth moved. An analogy is made between dropping a stone from a tower on the earth and dropping a rock from the mast of a ship. This analogy is advanced as support for the Aristotelian view, since the rock will fall straight down if the ship is still, but "will strike at that distance from the foot of the mast which the ship will have run during the time of fall" (p. 126) if the ship is moving (See Figure 2). Analogously, if the earth were rotating then the rock should fall well behind the tower; since it does not, the earth is still.

Salviati turns the analogy against the Aristotelian position. First, he brings up disparities between the base and target domains, such as differences in air and wind behavior. Simplicius considers these disparities and decides

that they do not invalidate the analogy. Having confirmed that the analogy is binding, Salviati administers the final stroke (pp. 144, 145):

Salviati: Now tell me: If the stone dropped from the top of the mast when the ship was sailing rapidly fell in exactly the same place on the ship to which it fell when the ship was standing still, what use could you make of this falling with regard to determining whether the vessel stood still or moved?

Simplicius: Absolutely none;...

Salviati: Anyone who does will find that the experiment shows . . . that the stone always falls in the same place on the ship, whether the ship is standing still or moving with any speed you please. Therefore, the same cause holding good on the earth as on the ship, nothing can be inferred about the earth's motion or rest from the stone falling always perpendicularly to the foot of the tower.

Having mapped ship onto earth, mast onto tower, and rock onto stone, Galileo's scientist can then carry across the relational structure, including a rather surprising set of relationships. Since the rock falls straight from mast to

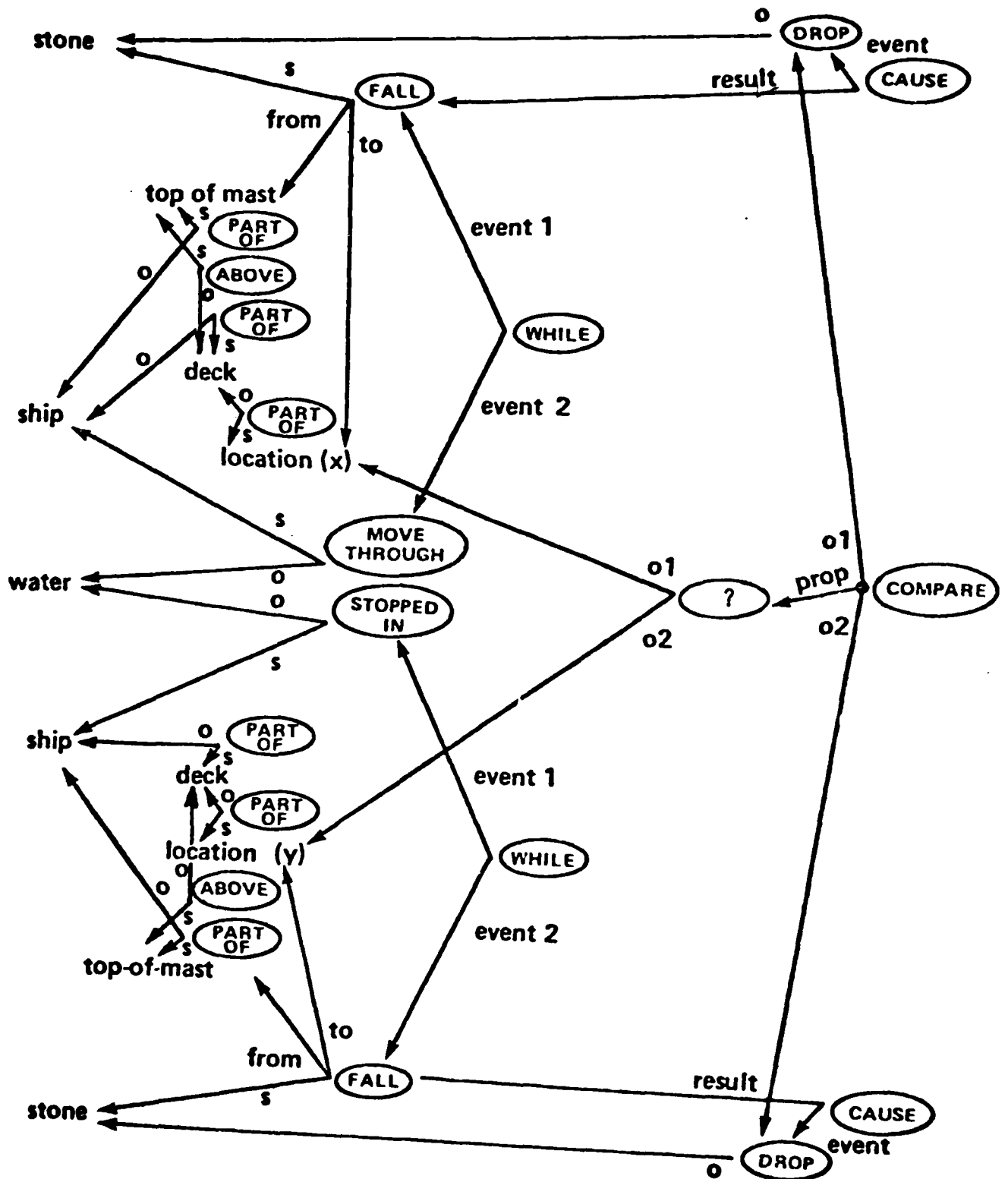
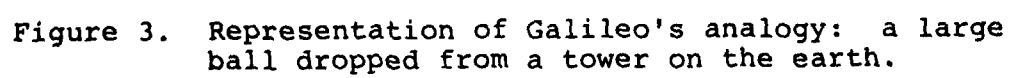


Figure 2. Representation of Galileo's analogy: a rock dropped from the mast of a ship.



deck even when the ship is moving, the straight fall of the stone from the tower cannot be used as evidence for a motionless earth. Galileo can be seen to use an active process of setting up the critical comparison, explicating the object-correspondences and relational identities clearly, and then deriving new predictions by mapping hitherto unsuspected relationships from the base to the target. The new relationship mapped here (that the rock falls to the same spot regardless of the motion of the carrier) is particularly striking in that it is the opposite of what was initially believed, at least by Simplicius. Yet the rules of analogical mappings are such that, unless there is a principled reason to exempt a given predicate, it must be mapped if it belongs to the system.

Evidence for structure-mapping. There is some preliminary empirical support for the claim that structure-mapping is central in systems analogies. As part of a larger study, we asked people to write out their interpretations of various metaphorical comparisons. The structure-mapping hypothesis predicts that, at least for explanatory analogies, people will seek structured sets of relations to map between the two domains. Thus there should be a general pattern: (1) relationships will predominate over attributes in the interpretations; (2) if a large overlapping

set of relations is found, subjects will rate the comparison as apt; (3) if no satisfactory set of relations is found, then target and base will be treated as holistic objects, and attributes of the base will be carried across; and (4) in this case the aptness rating will be low.

Subjects' interpretations of the statement, "An atom is like the solar system." are shown in Table 1. It can be seen that most of the responses conform to the predictions of the structure-mapping hypothesis. Some subjects explicitly mention either the object correspondences or the carry-over of identical relationships from the solar system to the atom or both. Other subjects express the structure-mapping by abstracting from both systems a set of general objects with the desired common set of structural relationships. This description is then applicable to either the base or the target domain. In all, nine of the ten interpretations focus on relations between objects and not object attributes.⁴

Predictions (2) and (3) are also borne out: We asked the subjects to rate the subjective aptness of the comparison -- how "apt, interesting, informative" the comparison was. The aptness ratings were highest (mean of 3.8) when subjects performed structure-mappings and focused on relationships in their interpretations. The one subject (#18) who focused on object-attributes gave the comparison the lowest possible

Table 1
Interpretation of Explanatory Analogies
An atom is like the solar system

Kind of response	Responses	Aptness ratings
<u>Standard SM: Mappings</u> between different objects reveal identical relation- ships	(15) Planets orbit the sun held in orbit by gravity like the electrons orbit the nucleus held by gravity. Both consist of larger nuclear body and smaller orbiting satellites. (17) This metaphor brings to mind a central object with peripheral objects moving around it. Atoms have protons and electrons moving around a neutron.	4 3
<u>Abbreviated SM: Mapping</u> between different objects; identity of relations assumed	(12) Electrons = planets; Paths = orbits; Sun = neutron; nice small unit. (21) Central nucleus, electrons versus planets. Particles, meteors, satellites revolving around.	4 3
<u>Generalized SM: Objects are</u> abstracted to reveal identical relationships	(13) Like the solar system, the atom is a collection of items 'revolving' around a central mass. (14) Both have a nucleus with smaller items in orbit. (16) Most of the mass of the atom is contained in the center, which by complicated forces holds the outermost units to stay with the center.	5 4 2

Table 1 - continued

An atom is like the solar system

Kind of response	Responses	Aptness ratings
<u>Generalized SM:</u>		
	(19) Implies that the atom has a large, dense attractive center, with smaller particles travelling in orbits around it.	5
	(20) In the atom there are things which revolve around a central nucleus in an orderly fashion.	4
<u>Mean aptness rating</u>		
for structure-mappings		3.8
<u>Holistic comparison:</u>	(18) An atom is like the solar system in that both can be broken down into parts. Both need to be whole to be really effective.	1
Object mappings not delineated		
<u>Mean aptness rating</u>		
for holistic comparison		1.0

aptness rating. This suggests that subjects may feel that a comparison for which only attributes can be found in common is a poor one.

The atom-solar system analogy is a familiar one, and subjects' interpretations might have been influenced by previous exposure to the model. Table 2 shows the subjects' interpretations of a less familiar analogy, Newton's analogy mentioned earlier: "The orbit of a planet is like the trajectory of a ball." Subjects were more critical of this unfamiliar analogy; three of the ten subjects focused on flaws in the comparison, whereas no subject did so for the atom-solar system analogy. However, the basic patterns are still in conformity with the structure-mapping hypothesis. First, interobject relationships appear in the interpretations more often than object attributes. Six out of ten subjects produced structure-mapping interpretations here (as compared to nine out of ten for the atom-solar system analogy). Notice that in order to produce relational interpretations, subjects had to invoke objects - the sun, in the case of the planet, and the earth, in the case of the ball - that were not mentioned in the original analogy. People are apparently willing to go beyond the information given in the original statement in order to find an appropriate system of interrelated objects. Second, in cases when attributes do

figure in the interpretations, the rated aptness is low. The mean aptness rating given a holistic interpretation was 1.5, as compared with a mean rating of 3.7 given a structure-mapping interpretation. In the larger version of this study, we found a positive correlation between the independently rated relationality of subjects' interpretations of analogies and their aptness ratings for those analogies, and a nonsignificant negative trend in the correlation between attributionality and aptness. This supports the claim that subjects seek structural interpretations of explanatory analogies, and accept holistic attribute-matches only as a less preferred alternative.

These parallels lend support to the claim that structure-mapping is a general processing heuristic available to subjects in a scientific context. As further support for the structure-mapping hypothesis, it appears that the finding of a set of mappable relations may be more important in determining perceived aptness than whether the subject ultimately accepts or rejects the analogy. The one structure-mapping subject who was critical of the second analogy (#15) nevertheless gave the comparison an aptness rating of 3. This rating is slightly lower than the ratings of the other structure-mapping subjects, but higher than the ratings given by holistic subjects, whether or not they

Table 2
Interpretations of Explanatory Analogies

The orbit of a planet is like the trajectory of a ball

Kind of response	Responses	Aptness ratings
<u>Standard SM: Emphasis on matches</u>	(13) Like the trajectory of a ball, the orbit of a planet is the path is makes through space under influence of gravity. (16) A planet's path is subject to the attraction of the star and describes the same path a ball would if all quantities are appropriately scaled. (19) Both planets and balls are spherical masses. Implies that planets travel through space in paths affected by gravity just as ball travels through air.	5 5 4
<u>Standard SM: Emphasis on differences.</u>	(15) The ball is influenced by the gravity of the planet it is on but the masses are so different that the ball is far too small and moving too slowly to overcome the gravity.	3
<u>Generalized SM: Objects abstracted to reveal identical relationships</u>	(17) This conveys to me a round object moving in relation to some fixed object.	3

Table 2 - continued

The orbit of a planet is like the trajectory of a ball

Kind of response	Responses	Aptness ratings
<u>Generalized SM: Continued</u>	(18) The orbit of a planet is like the trajectory of a ball in that both involve paths something follows. An orbit has a path it follows around a planet.	2
	(20) That the orbit of a planet refers to the path of a spherical object as a result of acceleration, gravity and velocity and the time elapsed.	2
<u>Mean aptness rating for structure-mappings</u>		3.4
<u>Holistic comparison:</u>	(12) Trajectory 1-way orbits come back to the starting point. They are both curved.	1
<u>Emphasis on matches</u>	(14) The orbit is a circuit--a function which repeats itself. The trajectory of a ball is not.	1
<u>Holistic comparison:</u>	(21) Orbit does not decay. Trajectory is limited and not complete.	
<u>Emphasis on differences</u>	Orbit is complete and repetitious. It is curved, powered by gravitation.	2
<u>Mean aptness rating for holistic comparison</u>		1.3

accepted the comparison. The aptness ratings given holistic interpretations were quite low, ranging from 1 to 2, regardless of whether similarities or differences were stressed. Thus, these preliminary results are consistent with the basic claim that complex explanatory analogies are understood primarily as structure-mappings.

Characteristics of Scientific Analogies

Given the structure-mapping description as a framework, we can pursue the further question of what makes a good scientific analogy. The most obvious criterion is whether the model is valid, i.e., whether the relations imported from the base are true in the target. However, validity, though clearly important, is the wrong place to start, because it ignores the systems properties of analogy. One does not judge an analogy by validating all the possible relational mappings from base to target. For example, in Galileo's earth/ship analogy, we do not attempt to map the ratio between the volume of the ship and the volume of the mast; it is clearly irrelevant, and whether it does or does not correspond to the ratio in the earth-tower system does not affect our judgement of the analogy. On the other hand, an analogy can be ruled out as untestable if, for example, it contains a self-contradiction, even if it has a large number of valid submappings. The point is not that validity is unimportant,

but that what counts as valid requires careful consideration. A science analogy must be seen as a system of mappings, not an undifferentiated set of predicates to be judged simply by numerosity.

Therefore, let us turn to the structural qualities of a good science analogy, holding validity constant for now. In this section I propose a set of structural considerations that can be used to characterize good science analogy. In order to show that these distinctions differentiate between useful and nonuseful analogy, I then apply them to different kinds of analogies, showing how good scientific uses of analogy differ from other uses of analogy.

The first consideration, which arises even before the analogy is defined, is base specificity. This refers to the degree to which the structure of the base is explicitly understood. The better analyzed the base, the clearer the candidate set of mappable relations will be. This is one reason that the base is usually a familiar domain; though familiarity is no guarantee. For example, sometimes in introductory chemistry texts, molecular bonding is explained by analogy with interpersonal attraction, e.g., "The lonely sodium ion searches for a compatible chloride ion." Interpersonal attraction is certainly familiar, but its rules are unfortunately unclear; so this analogy does not tell the

student precisely what to map from the base. At the other end of the spectrum are mathematical models, which use an exceedingly well-specified base. For example, given as base the real numbers with the relational operators of addition and multiplication, we can know exactly how these relationships apply. If we have $b_1 + b_2 = b_3$ in the base, then we know exactly what to predict for the corresponding target nodes: $t_1 + t_2 = t_3$.

The degree of base specificity imposes an obvious limit on the usefulness of an analogy, since the predicted target relations mirror the base relations. Therefore the predicted target structure cannot be better specified than the base structure. However, it can certainly be worse specified. It is perfectly possible to construct a poor analogy using a well-specified base. This brings us to the first internal-structure consideration, that of the clarity of the mapping.

Internal Structural Characteristics. The first and most fundamental structural consideration is clarity. The clarity of an analogy refers to the precision with which the mappings can be traced, i.e., exactly how the base nodes map onto the target nodes and which predicates get carried across. Any case in which it is unclear which base nodes map onto which target nodes violates clarity. One such violation occurs if

one base node maps to two or more relationally distinct target nodes (the one-to-many case) or if two or more relationally distinct base nodes map to the same target node.⁵ One variation of a many-to-one violation occurs when a base term is productively polysemous, with different senses entering into different relationships. Such an analogy is unfalsifiable, since any challenge can be met by a shift to the other relational framework. Since clarity is discussed at length later on, I will defer examples for now.

Strong explanatory analogy is characterized by a well-specified base domain and a clearly defined set of correspondences. There are other considerations as well. One important characteristic is richness: roughly, the quantity of predicates that are meant to be mapped. More precisely, the richness of an analogy is its predicate density: for a given set of nodes, the average number of predicates per node that can be plausibly mapped from base to target. Richness is defined independently of internal consistency; a set of predicates can all contribute to richness even if they involve contradictory mapping assumptions. Moreover, a predicate can contribute to richness even if it is false or does not possess a truth value in the target, as long as it has enough plausible appeal to be mapped. For example, affective relations can contribute to richness. Therefore, the richness

of an analogy, like its clarity, can be discussed without assessing its validity.

Next there are two considerations, abstractness and systematicity, that go beyond the number of predicates mapped to consider the kinds of predicates mapped. First is the abstractness of the mapping: where in the structural hierarchy the mapped predicates are found, i.e., whether they are attributes or relations, and if relations whether they are higher-order or lower-order relations (See Smith, in preparation). A relation among objects is a first-order relation. A relation among first-order relations is a second-order relation, etc. Since a higher-order relation is essentially the name of a system of relations, such a relation is very useful in conveying the overall structure of a domain. The greater the proportion of higher-order relations, the more abstract the mapping. Thus an extremely nonabstract mapping would be one in which attributes only were conveyed, and an extremely abstract mapping would be one conveying only higher-order relationships.

There is often a trade-off between abstractness and richness. Although one could in principle devise analogies with very large numbers of clear, high-level relational correspondences, in practice very rich analogies tend also to be fairly attributional, often sensory, and therefore nonabstract.

The next consideration goes beyond the hierarchical characteristics of the predicates to consider their interrelatedness. This is the systematicity of the mapping - the degree to which the relations mapped belong to a known mutually constraining conceptual system. An analogy in which separate or ad hoc relationships are mapped is less systematic than one in which a set of coherent, mutually constraining relationships are mapped. To be systematic, an analogy must include a fair number of abstract relations, since the constraints between lower-order predicates are structurally represented by higher-order (i.e., abstract) relations between those predicates. However, not all abstract analogies are systematic, since systematicity has the further stipulation of mutual constrainedness. A mapping is systematic to the degree that any given predicate can be predicted or derived from all the others.

To see the usefulness of this kind of structural redundancy, consider the Rutherford model, a highly systematic analogy. Here the mapped relationships - ATTRACTS (sun, planet), ORBITS AROUND (planet, sun), etc. - form a connected system, together with the abstract relationship INVERSE-SQUARE CENTRAL FORCE BETWEEN (sun, planet). Many of the lower-order relations could be predicted from this higher-order relation. To see the systematicity more clearly, consider the equations⁶

that sum up the relations between predicates in base and target.

$$F_{\text{grav}} = Gmm/r^2$$

maps into a corresponding target equation

$$F_{\text{elec}} = - qq'/r^2$$

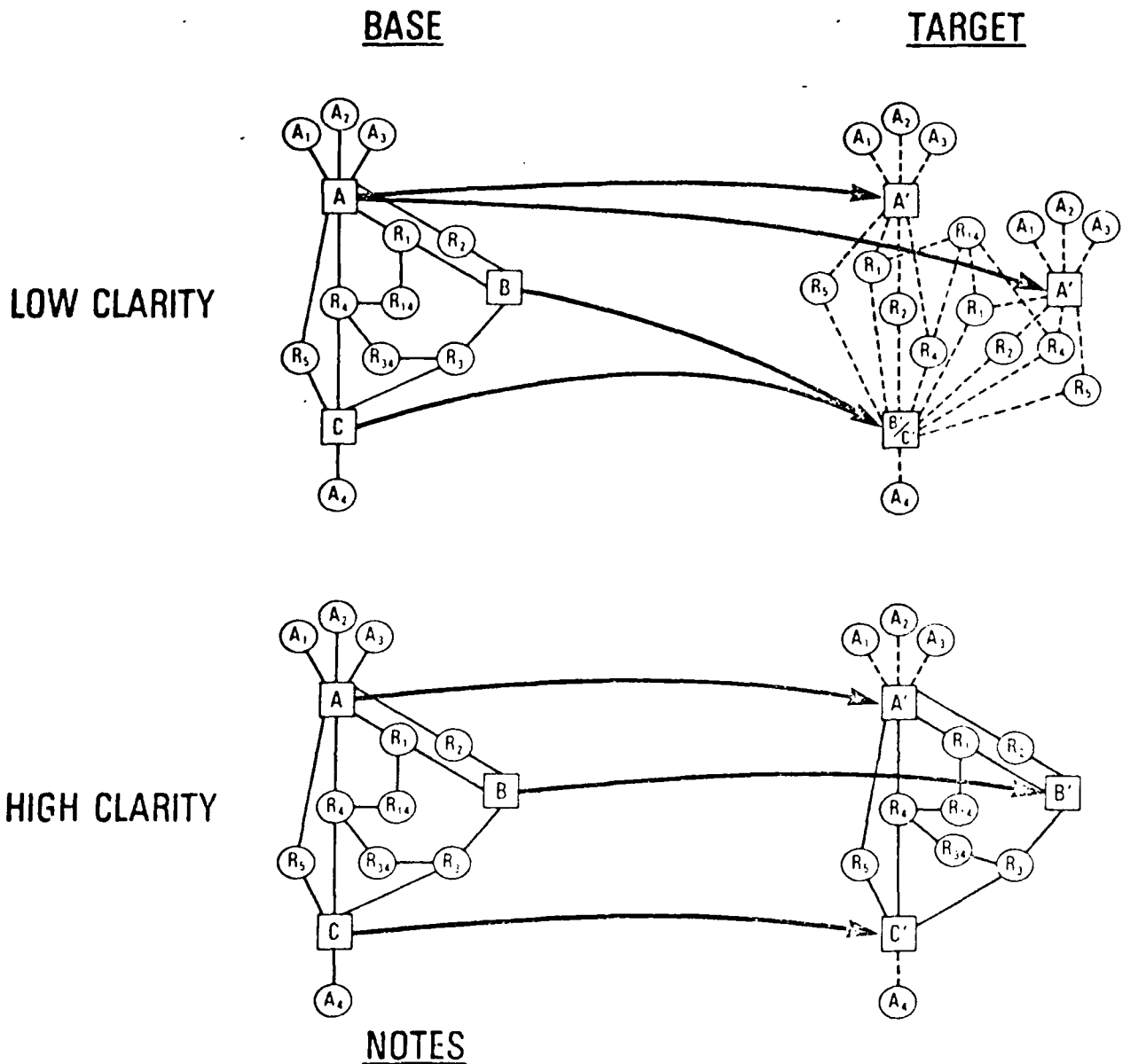
where m is the mass of the sun, m' the mass of the earth, q the charge on the proton, q' the charge on the electron, r the distance between the two objects, and F_{grav} and F_{elec} are the gravitational and electromagnetic forces. These equations embody a mutually constraining set of predicates. For example, knowing the electrical charges and resultant radial attraction, we can predict the distance between nucleus and electron.

To summarize, the list of important considerations starts before the analogy is really underway with base specificity: how well do we understand the base. Once the analogy is given, without stopping to assess its validity we can ask about its clarity: how well is the mapping specified. We can also ask about the structural issues of (1) richness - how many predicates are mapped for every target node; and more qualitatively, (2) abstractness - what hierarchical level are the mapped predicates from; and (3) systematicity - how much is each of the mapped predicates constrained by the others. Figure 4 shows in schematic form the structural

distinctions involved in clarity, richness, abstractness and systematicity. The figure largely recapitulates the text; however there are a few points to notice. First, in the low-clarity analogy the uncertainty as to how to map the predicates is greater the higher-order the relation, since the indeterminacies propagate. This fits with the intuition that an unclear mapping is difficult to formalize. Second, the clarity distinction is unique in that it affects the node-mappings; the other characteristics concern which predicates are mapped, given a particular correspondence among nodes.

Validity, exhaustiveness and transparency. So far we have not considered whether the mapping is valid. We have been concerned with the structure of an analogy's predictions, not with the correctness of the predictions. The clarity and base specificity of an analogy, and even its richness, systematicity and abstractness, can to some extent be discussed independently of whether the analogy holds. If we now consider the target validity of the predicates mapped, it is clear that, in general, not all the base relations will be valid in the target.

Given a mapping of the nodes of B into the nodes of T, the validity of a base predicate refers to its truth value or correctness as applied in T. (It is assumed to be valid in B.)



NOTES

B : (base): node B.

B' : (target): node to which the base node B is mapped.

A₁ : attribute of node.

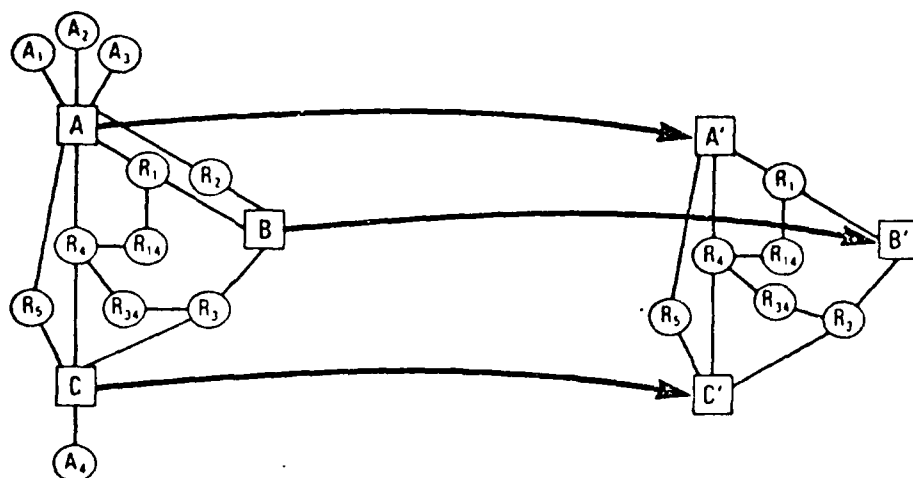
R₁ : relation between two nodes.

R_{1,2} : relation between two relations.

--- : uncertain application of the given attribute or relation.

Figure 4. Schematic depiction of structural distinctions involved in clarity, richness, abstractness and systematicity.

LOW RICHNESS



HIGH RICHNESS

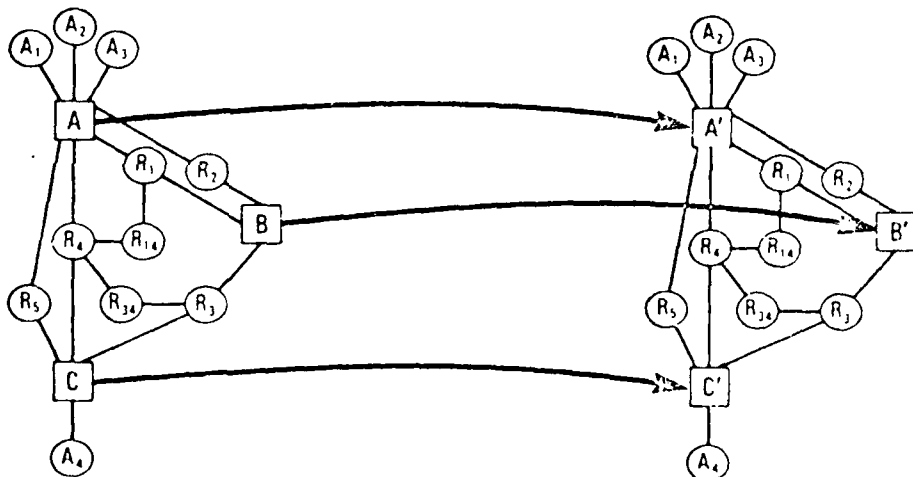
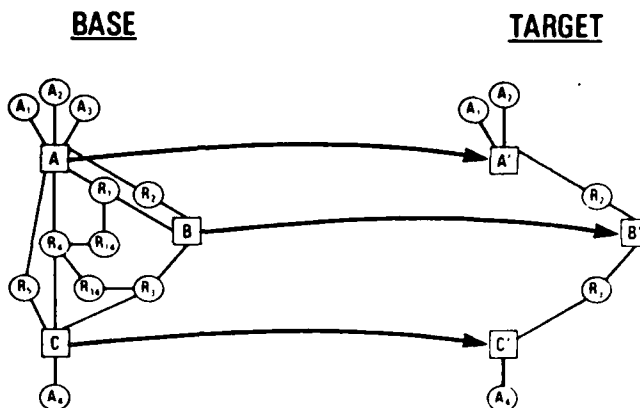
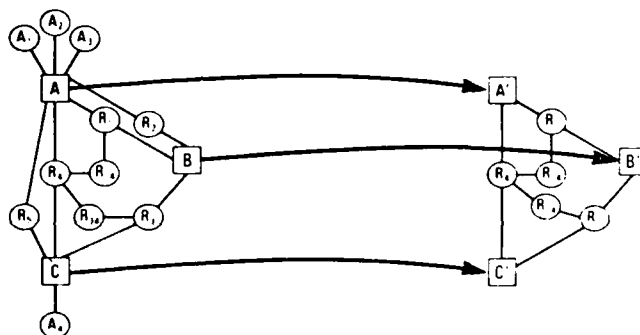


Figure 4 - continued

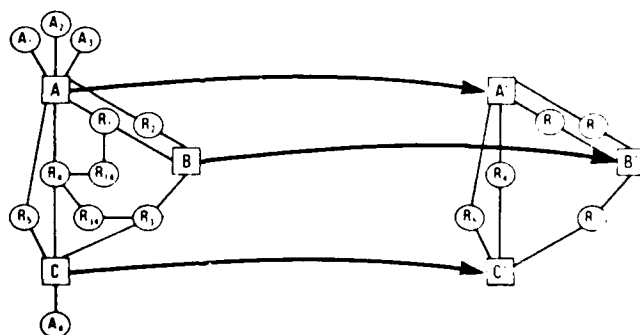
LOW SYSTEMATICITY-
LOW ABSTRACTNESS



HIGH SYSTEMATICITY-
HIGH ABSTRACTNESS



LOW SYSTEMATICITY-
MODERATE ABSTRACTNESS



NUMBER OF
PREDICATES
MAPPED

SECOND-ORDER RELATIONS	FIRST-ORDER RELATIONS	ATTRIBUTES
0	2	3
2	3	0
0	5	0

Figure 4 - continued

The validity of each predicate can be decided separately, and independently of the clarity of the analogy as a whole. However, for the overall validity of an analogy to be high requires that the mapping be internally consistent. Overall validity will be low if the correspondence between nodes and the resulting carryover of relationships from the base domain generates self-contradictions in the target.

The target exhaustiveness of the analogy refers to the proportion of the target relationships the model accounts for; that is, how many of the significant relational predicates in the target domain can be mapped from the base. In mathematical terms, if the target exhaustiveness were as large as possible, the target domain would be covered; all of its relational predicates would be accounted for, and the mapping of relationships would be onto. Base exhaustiveness refers to the extent to which the structure of the base domain is applicable to the target domain: that is, to the proportion of the total number of relational predicates in the base that can be applied in the target domain. There is no special mathematical term here, since a function $M:B \rightarrow T$ is normally assumed to utilize all elements of B. Target exhaustiveness and base exhaustiveness are complementary aspects of the completeness of a given model.

Since most mappings of interest are non-exhaustive in both the base and target, it is sometimes useful, particularly in educational uses, to consider the transparency of the analogy: the ease with which it can be decided just which predicates from the base domain are to be applied in the target domain. Transparency is concerned with whether the separation into valid and nonvalid predicates is a natural one. It is greatest when there is a structural partition of some kind between the base relations that carry over and those that do not.

Scope: an extrinsic consideration. So far the discussion has focused on the structural properties of the domains and of the mapping. We have discussed five structural considerations - base specificity, clarity, abstractness, richness, and systematicity - that can precede a validity check; as well as three characteristics - base and target exhaustiveness and transparency - that arise after the validity of the imported predicates is assessed. There is also an important external consideration: the scope of a model. This refers to the number of different cases to which the model validly applies. For example, the solar system model works reasonably well for the hydrogen atom, but less well for heavier atoms; and it is simply not applied at the molecular level. The scope of an analogy is in principle

unrelated to its internal structural characteristics. However, in practice, it is hard to design an analogy that conveys a high density of predicates over a broad range of different target instances without allowing the definitions of objects and predicates to slide about. Thus there tends to be a three-way trade-off between scope, richness and clarity.

The next portion of the paper is devoted to using these structural distinctions to arrive at a characterization of what makes a good explanatory-predictive analogy. Two contrasts are helpful. The first is between explanatory analogy - analogy intended to explain and predict - and expressive analogy - analogy intended to evoke or describe. The second contrast is between good explanatory analogy and poor explanatory analogy. The structural considerations discussed in the last section are applied first, to contrasting pairs of expressive vs. explanatory analogies; and second, to contrasting pairs of good and bad explanatory analogies.

Explanatory versus expressive analogy. Let us begin by comparing Galileo's earth/ship explanatory analogy, given above, with an expressive analogy from T.S. Eliot's The Hollow Men:

"Leaning together
Headpiece filled with straw. Alas!
Our dried voices, when
We whisper together
Are quiet and meaningless
As wind in dried grass
Or rats' feet over broken glass
In our dry cellar."

Both Eliot's metaphor and Galileo's analogy are nonliteral comparisons between pairs of domains; both involve object mappings and relational carryover. Galileo's analogy maps ship to earth, mast to tower, and rock to stone. Eliot's analogy maps dried grass and/or rat's feet to voices. Yet there are differences. To begin with, the explanatory analogy seems less sensuous. In Galileo's ship-earth comparison the shape of the ship, its color, its smell, its barnacle-encrusted texture are stripped from the mapping, leaving only relationships - the perpendicular vertical relationship of mast to ship, the motion of the ship relative to the medium, and so on. By contrast, in Eliot's lines, the sensory attributes - the dry feel, the rustling sounds, the bleached colors - of the dried grasses, the broken glass, and the rat's feet are paramount. Object attributes are more important, and higher-order relations less important, in

Eliot's analogy than in Galileo's. Thus, Eliot's analogy is richer and less abstract than Galileo's.

Further, in the Eliot example, base specificity and clarity are conspicuously lower. There are a great many connective relationships between hollowness/stuffing/straw/dry grass/dry cellars/rats' feet and so on, which cannot easily be spelled out. A particularly straightforward suspension of the clarity principle is the treatment of the target node voices, which is given two different analogical comparisons: wind and rats' feet. These interact with each other and with the passage to produce a rich web of connections. Precisely defining these relationships does not seem to be important or even appropriate; they are intended to be appreciated without too much analysis.

In contrast, the Galileo example is high in base specificity; the predicates in the ship (base) domain are laid out clearly. It is also low in richness - there are almost no attribute mappings, for example. It is high in clarity. There are no disjunctive mappings; every base node maps unambiguously onto a target node, so that the set of potentially mappable relations is clear. The analogy as stated is only moderately systematic; in general a given target (or base) predicate is not strongly constrained by the other target (or base) predicates.

Notice that validity alone - that is, the validity of all the base predicates as applied in the target - is not as good a differentiator between explanatory and expressive analogy as are the structural characteristics. Both kinds of analogy include invalid as well as valid potential predicate-mappings; for example, Galileo worries about the fact that the ship's motion is artificially caused while the earth's motion is natural.

Rutherford's atom/solar system analogy shows similar structural characteristics to Galileo's analogy. It is high in clarity, as Figure 1 shows: both the node mappings and the relational carryover are unambiguous. It is high in abstractness and low in richness: one-place predicates, such as HOT (sun), YELLOW (sun) are not preserved. Instead the mapping focuses on more abstract relational predicates, such as ATTRACTS, which form a systematic set.

As in Galileo's analogy, there are nonvalid as well as valid potential predicate-mappings. Here a major nonidentity lies in the nature of the forces. In the solar system, the force relationships are gravitational and positive: the planets mutually attract one another, just as they attract and are attracted by the sun. But the electromagnetic force in the atom is attractive or repulsive depending on whether the particles are of unlike or like charge. Thus if we decompose

gravity into its component relationships of ATTRACTS (sun, planet) and ATTRACTS (planet, planet) and electromagnetism into ATTRACTS (nucleus, electron) and REPELS (electron, electron), we find that only the first can be validly mapped; the second is nonvalid in the target. This model, then, is nonexhaustive: not all the base relations are valid in the target. As was argued earlier, this is almost surely true of all complex analogies. In any case, it is not a serious problem for Rutherford's analogy, because the difference is fairly transparent.

These contrasts suggest some possible structural differences between expressive and explanatory analogy. The first of these lies in the relative values placed on richness versus clarity in the predicate structure. In expressive analogy there may be greater value in richness - in the sheer number and density of relationships conveyed - than in ensuring that all the mappings are clear and consistent. There may be greater emphasis on clarity in explanatory analogy. These contrasts also suggest a difference in abstractness: in the kinds of predicates mapped, aside from the sheer density of predicates and the clarity of the mapping. It may be that surface sensory attributes figure more strongly in expressive analogies than in explanatory analogies, so that their abstractness will in general be

lower. The suggestion, then, is that in expressive analogy, a rich collection of associations is valued; while in explanatory analogy, an abstract, well-clarified, coherent system of relations is valued. This fits with Boyd's (1979) observation that for science analogies, further explication and analysis is taken as a community enterprise, while literary metaphors, in contrast, are treated as wholes. The dissection of literary metaphors is considered an academic exercise best left to critics and teachers of writing, and not a necessary part of the actual enterprise of writing.

However, literary metaphors differ. Before drawing conclusions, we should consider the work of Shakespeare, Donne and others, whose expressive analogies are relatively clear and abstract. For example, Shakespeare's "It is the east, and Juliet is the sun" comparison is not meant to convey the lowest-level sensory attributes; it does not lead us to assume that Juliet is hot and gaseous, nor yellow in color. It is primarily relationships that are conveyed: the spatial relation of Juliet appearing above the window as the sun rises above the eastern horizon, and the affective relations of her causing hope and gladness in Romeo as the sun causes them in earthly creatures. Thus the mapping is somewhat abstract. Locally, its clarity is fairly high as well: the sun maps to Juliet, the window to the horizon, and so on. However, the passage goes on to shift the set of mappings:

But, soft! what light through yonder window
breaks?

It is the east, and Juliet is the sun! -
Arise, fair sun, and kill the envious moon,
Who is already sick and pale with grief,
That thou her maid art far more fair than she:
Be not her maid, since she is envious;
Her vestal livery is but sick and green,
And none but fools do wear it; cast it off. -

(Romeo and Juliet, Act II, Scene 1)

The target system remains Juliet and the moon, but the base shifts from the sun and moon to maid and mistress. The mapped relationship between Juliet and the moon thus shifts from CAUSE (RISE (Juliet), DISAPPEAR (moon)), on the sun-moon based analogy to ENVY (moon, Juliet) and CAST OFF (Juliet, appearance (SUCH THAT (BELONG (moon, appearance)))), based on the maid-mistress analogy. Thus in spite of the local clarity and a certain degree of abstractness, systematicity and large-scale clarity are not preserved here.⁷

Thus, explanatory analogy may differ from expressive analogy in any or all of four ways: (1) abstractness: conveying higher-level relations among objects, as opposed to object-attributes and first-order relations; (2) richness: conveying fewer predicates, whether attributes

or relations; (3) clarity: being more consistent and more liable to clarification; (4) systematicity: being more constrained to utilize a single coherent system of relations, and (5) base specificity: tending to utilize very well-understood domains as the base domains.

To test for some of these hypothesized structural differences between scientific and literary analogy, we asked subjects to rate scientific and literary metaphors for richness and for clarity. The metaphors included twenty scientific analogies and twenty literary analogies. Ten of the science analogies were, in the experimenters' opinion, good analogies. The other ten were poor analogies in use either currently or historically. The literary comparisons were similarly chosen to include ten good and ten poor analogies. Samples of the materials are shown in Table 3.

As a check on these assignments, half of the subjects rated the metaphors for scientific explanatory value, and the other half rated them for literary expressiveness. Table 4 shows the mean richness and clarity ratings for the four a priori classes of metaphor; good science, poor science, good literature and poor literature. The subjects' mean ratings of scientific explanatory value and literary expressiveness are also shown.

Table 3

Sample comparisons of different kinds showing subject's mean ratings of scientific explanatory value (S.E.V.), literary expressiveness (L.E.), clarity and richness

Comparisons	S.E.V.	L.E.	Clarity	Richness
<u>Good Science</u>				
An electric circuit is like a system of water flowing through pipes; the water is the current; the reservoir is like the power source, the dams are like resistors, the potential energy in the stored water is like the voltage in the circuit, and so on.	4.5	4.4	4.7	3.2
Echoes are like big balls of air that bounce off walls.	3.3	2.8	4.1	2.3
<u>Good Literature</u>				
With a light soul to cover him, this jolly fellow lives in fun, and sorrow and care blow over him like a breeze over a warm comforter.	2.7	3.7	2.9	3.0
In this mortal state of imperfection, fig leaves are as necessary for our minds as for our bodies; it is as indecent to show all we think as all we have.	2.7	4.6	4.0	3.9

Table 3 - continued

Sample comparisons of different kinds showing subject's mean ratings of scientific explanatory value (S.E.V.), literary expressiveness (.L.E.), clarity and richness

Comparison	S.E.V.	L.E.	Clarity	Richness
<u>Poor Science</u>				
Forging a rare metal is like planting a garden. Seeds of metal are planted over a furnace within flasks, with a low fire beneath them, and are kept warm day and night. From these little eggs there will be produced enormous shining chicks.	1.9	2.4	2.6	2.7
In a marble statue, marble is the matter and the shape given by the sculptor is the form; similarly, in a calm sea, water is the matter and smoothness is the form. Only because of its form does matter become a thing.	2.8	2.9	3.5	3.6
<u>Poor Literature</u>				
Money is like an arm or a leg -- use it or lose it.	3.3	2.9	4.1	2.1
Any large company must be a changing company. You can't sink anything in concrete because that attitude will slowly seep through to all levels of the organization and all of a sudden you're lying dead in the water.	2.4	2.5	2.7	2.3

Table 4

Mean ratings of different qualities for different
classes of metaphors

A priori classifications	Clarity	Richness	Scientific	Literary
			explanatory value	expressiveness
Good science	3.87	2.42	3.53	3.22
Poor science	3.30	2.92	2.94	2.77
Good literature	3.41	3.42	2.91	3.35
Poor literature	3.52	2.76	2.93	2.94

As predicted, the scientific and literary analogies have an almost opposite relationship to richness and clarity. For science analogies, good exemplars are rated higher in clarity than poor exemplars. Clarity ratings for good literary metaphors are essentially the same as ratings for poor literary metaphors. In contrast, richness ratings for science analogies are lower for good exemplars than for poor exemplars. Richness in literary comparisons is higher for good exemplars than for poor exemplars. This pattern fits with the prediction: clarity must be present in good explanatory analogy, and may be present in good expressive analogy. On the other hand, richness contributes strongly to the goodness of expressive analogy, but not to explanatory analogy.

These patterns are based on our a priori judgments of literary and scientific goodness. We also correlated the subjects' own ratings of scientific explanatory value and literary expressiveness with their ratings of richness and clarity. The correlations show the same patterns as the a priori analyses, except that clarity appears more important for literary goodness. Judgments of clarity correlate strongly and positively with judgments of scientific explanatory value ($r = .77$; $p < .0001$ F , two-tailed) and also with judgements of literary expressiveness ($r = .68$; $p < .0001$).

Richness is not correlated with scientific explanatory value ($r = .16$; $p < .32$), but is correlated with literary expressiveness ($r = .42$; $p < .005$).

Thus, our subjects found clarity important in judging the goodness of both expressive and explanatory analogies. Richness, however contributed only to literary expressiveness and not to explanatory value. Thus our expectation that clarity should be important in science analogies was confirmed, as was the expectation that richness would contribute only to literary expressiveness. The role of clarity in expressive analogies is not certain. It may be that both richness and clarity are desirable in expressive analogy, when possible, and that our materials did not force subjects to choose. (We did not include any literary metaphors as rich and low in clarity as Eliot's example.) Further research may reveal exactly how these considerations interact.

Good versus poor explanatory analogies. The analogies of the alchemists are often cited as quintessential instances of poor explanatory analogy. A careful examination of one of these analogies may reveal structural reasons for its inefficacy as a predictive model. Consider this example from the alchemist and healer Paracelsus, writing within a century of Kepler and Galileo:

... what, then, is the short and easy way whereby Sol (gold) and Luna (silver) can be made? The answer is this: After you have made heaven, or the sphere of Saturn, with its life to run over the earth, place it on all the planets so that the portion of Luna may be the smallest. Let all run until heaven or Saturn has entirely disappeared. Then all those planets will remain dead with their old corruptible bodies, having meanwhile obtained another new, perfect and incorruptible body. That body is the spirit of heaven. From it these planets again receive a body and life and live as before. Take this body from the life and the earth. Keep it. It is Sol and Luna.

(Paracelsus, ca 1530, quoted in Jaffe, 1976, p. 23)

The first thing one notices about this model is its air of mystery. Although it uses a generally known standard set of correspondences - e.g., Sol/gold; Luna/silver; Saturn/lead - these do not lead to a clear predictive model. Why? One reason is the lack of clarity in the basic object-mappings, manifested, for example, in the interchangeability of "heaven", "the sphere of Saturn" and "Saturn". These comprise a disjunctive pair of base nodes like Eliot's "rat's

feet/broken glass" that results in an indeterminate mapping.

A second reason for the nonpredictive quality of this analogy is its lack of systematicity. Although the base domain is the solar system (including earth's moon and excluding then-unknown planets), the relations invoked are not the set of canonical relations in the solar system, even as known at the time. Instead of carrying a system of base relationships into the target, Paracelsus applies new relations over the mapped objects: e.g., "PLACE ON (heaven, all the planets)"; "MAKE RUN OVER (heaven, earth)". Even if these predicates had unambiguous interpretations, they are not part of an existing connected system of relations in the base and therefore they supply no mutual constraints. The solar system predicates that do figure in the analogy are only the object-attributes of the heavenly bodies; yellow color, for example, maps from Sol onto gold. The structural relationships among the objects are not utilized. In contrast, Rutherford's atom/solar system model ignores the object attributes in the solar system and focuses on the systematic set of relationships among sun and planets. Thus, even though Paracelsus uses the same base domain as Rutherford, the nonsystematic nature of the mappings, and more fundamentally the lack of clarity, render this model useless for prediction.

A more modern example of unclarified analogy is Freud's (1973; reprinted from 1955) discussion of anal-eroticism, in which it is claimed that, in unconscious thought, the concepts of feces (money, gift), baby and penis are often treated "as if they were equivalent and could replace one another freely" (1973, p. 86). The case for this correspondence includes linguistic evidence: the phrase "to give someone a baby", showing the correspondence between babies and gifts; phenomenological evidence that feces are the infant's first gift, and that money, as a later gift, comes to be equated with feces; and evidence from mapping attributes and first-order relations, such as that feces, penis and baby are all solid bodies that forcibly enter or leave through a membranous passage.

There is not space here to fully analyze this analogy, except to state that it is fundamentally lacking in clarity. Since there are five corresponding objects, any of whose attributes and relations can be mapped across to any of the others, the predictions are extremely fluid. One can shift around among the several object mappings to accommodate many disparate phenomena.⁸

Sound waves and water waves. An example of a fairly nontransparent model that is still systematic enough to be useful is the analogy between sound waves and water waves.

This is a useful analogy, which captures much of the behavior of sound, but it has the disadvantage that the invalid predicates are difficult to separate structurally from the valid predicates. Sound waves are longitudinal (back and forth) compression waves, while water ripples are transverse (up and down) waves. Thus there are limits on both base-exhaustiveness and target-exhaustiveness. More importantly, these limits do not appear to be transparent. Ideally, a person using this analogy should carry over the cyclic periodicity and the notion of a travelling wave front, but not the orientation of the cyclical motion relative to the direction of motion of the wave through the medium. Yet there is no clear structural partition between the fact of cyclic motion through a medium and the nature of that motion. This lack of transparency predicts that people will frequently overmap, and think of sound waves as transverse.

In a study designed to test this prediction, we gave 36 college students instructions to "draw a diagram of a sound wave". Transverse waves of some kind were produced by 72% of the subjects. Another 11% of the subjects produced general concentric waves, which can be counted as correct. Only one subject produced a clear longitudinal compression wave. Accuracy was limited even in recognition. Asked to choose the best depiction of a sound wave from a set of five figures, 64%

of the subjects still chose transverse waves. Only 31% correctly chose general concentric waves, and only two subjects, or .06%, chose clearly longitudinal waves. This comparison, then, although in principle high in clarity, abstractness and systematicity, seems to lack transparency. Just how this nontransparency affects learnability and accessibility of a science analogy remains to be investigated.

Expert versus Naive Models in Science

We have tentatively arrived at a characterization of explanatory analogies as higher in clarity, abstractness, systematicity and base specificity, and lower in richness than expressive analogies. Are people's naive models of science more like explanatory analogies or expressive analogies? We have interviewed people concerning natural phenomena such as evaporation processes (Gentner & Collins, in preparation). In the protocols of fairly naive subjects, we find that questions are often answered by local explanations drawn by analogy from core examples. For example, the core example of evaporation for some of our subjects is a boiling tea kettle. To the question "Why can you see your breath on a cold day?" one such subject replied "When the hot air that you breathe meets with the cold air of the atmosphere, it will tend to vaporize almost like steam from a kettle, which of course can be seen." In a later question concerning the rate of evaporation of hot

water in a refrigerator, he again applied the tea kettle analogy, stating ". . .hot water will only evaporate if it is sufficiently heated". He at first concluded that the water, not being heated, would not evaporate; rather, water would condense from the refrigerator into the pan. He later modified this conclusion; still, the original position was fairly striking.

The tea-kettle analogy leads people, at least initially, to hold that evaporation depends on the external application of heat, so that the water is hotter than its surrounding air. These same people may have other core examples in which evaporation occurs without such a temperature differential, as in the overnight disappearance of a mud puddle. Yet these examples are not readily brought together. Instead the person makes new predictions according to the core example with (apparently) the closest surface resemblance to the case at hand. There seems to be a desire for literal similarity (i.e., overlap of attributes as well as relations) in naive subjects. Other examples are discussed by Collins, Stevens, and Brown (1979) and by Collins, Stevens and Goldin (1979). These core examples are related to the phenomenal primitives that Di Sessa (personal communication, October, 1979) finds in his naive physics subjects.

Our expert subjects invoke fewer core examples. Individual examples are extended further, with the help of global explanatory notions such as temperature or molecular motion. Thus we speculate that a difference between experts and novices in a scientific domain is that the expert has an abstract global model with broad scope, while the novice has a pastiche of rich, only locally useful models. The experts' global models have a limited set of general operations and relationships: for example, "relative degree of molecular motion". In contrast, the novices' systems have a great many locally applicable predicates, such as "in sun/out of sun". Finally, the experts' models appear more systematic; the degree of cross-constrainedness among predicates is higher. Thus, naive models of science appear more like expressive analogies than do expert models.

Development of models. The differences between naive and expert explanatory analogies raise the question of how a scientific analogy develops historically. Do analogies begin rich and unclarified, so that they require pruning and clarification to become good explanatory analogies, or do they start life sparse, so that they require elaboration to become richer analogies? The differences between the models used by expert and naive modellers suggest that analogies start rich and become abstract only with effort. If this is true, the

process by which an initially rich comparison is developed into a workable model must consist in part of predicate-stripping, whereby nonessential predicates are removed from the comparison. Some predicates of the base that were initially mapped from base to target are no longer carried across. This heuristic would act to pare down the comparison from a large undifferentiated set of commonalities to a small number of structural identities.

An apocryphal example of the historical stripping of extraneous predicates occurs in the analogy between sound, light and water waves (discussed in Hesse, 1966, pp. 11-99). The correspondence may initially be perceived in terms of a loose collection of similar properties, events and relationships, such as "produced by striking water, etc./produced by striking gongs, violins, etc.," "produced by striking match into flame, etc." and "height of wave/loudness/brightness." This list of similarities is gradually supplanted by a set of abstract predicates that results from further decomposing the wave model and stripping away the domain-specific predicates. At this level of analysis, it can be seen that there is a set of identical predicates that holds for light as well as for sound and water: for example, "WAVE AMPLITUDE" or "WAVE FREQUENCY". Once the nonidentical predicates are removed, the abstract

relational identities are clear. The endpoint of this process of explication and predicate-stripping is often a mathematical model, which represents an extreme of abstractness, base specificity and systematicity.

This has led some scientists and philosophers to argue that true science should consist of formal mathematical theories, and that mechanical analogies are mere way-stations on the way to mathematical models. Hesse (1966, pp. 7-56) describes a classic debate of the early 1900's. Duhem, the French physicist and philosopher, held that analogical models are inferior to mathematical theories, being fundamentally incoherent, superficial and illogical. The English physicist Campbell, who defended analogical models, argued that: (1) to be intellectually satisfying a theory must provide a causal explanation, not a mere formal description; and (2) a mechanical analogy is a continual source of new predictions, in addition to those already understood well enough to be formalized. (Of course, new predictions can also be derived from a mathematical formulation; however they are often not the same predictions.)

This is not the place to resolve this controversy. However, Hesse, continuing the wave example, points out that even to apply the equation for water ripples

$$y = a \sin 2\pi f x$$

(where y is the height of the water at point x , a is the maximum height or amplitude of the ripples, and f is the frequency) to the case of sound and light, it is necessary to interpret y , a , and f for sound and for light. That is, even for a mathematical model, the usefulness of the theory depends on a mapping between terms in the base domain and terms in the target domain.

Psychologically, one suspects that both kinds of models are useful in generating predictions. Further, possessing two different models of a target area may well be better than having only one, particularly if the interrelationships between the models are known and utilized (Brown & Burton, 1975). A physical analogy can provide a contrasting model against which to check the solutions derived from a formal model, or it can provide the initial parsing of a problem (e.g., deKleer, 1977; Larkin, J.H., 1977; Larkin, K.M., 1978; Reif, in press). One may alternately work to strip a mechanical analogy of its inessentials and reduce it to a purely abstract statement, while at other times using it as a full blown analogy to derive new mechanically-inspired predictions.

Comparison with Other Current Views

Although very little work has been done on complex metaphor and analogy, some of the recent work on the

psychology of metaphor and analogy is relevant here. First, there are some very interesting treatments of analogy in which representation is not discussed explicitly, either because the focus is on some relatively separable aspect of metaphor, such as metaphor as speech act (e.g., Searle, 1979), or because the emphasis is on empirical findings or phenomenology (e.g., Gick & Holyoak, in press; Hoffman, 1980; Johnson-Laird, 1980; Lakoff & Johnson, in press; Verbrugge & McCarrell, 1977).

Most of the psychological theorizing about representation of metaphor has been geared towards holistic object comparisons. For example, in Ortony's (1979) salience imbalance theory, the base and target domains are represented as lists of attributes ordered by salience. A nonliteral comparison in this system consists of finding a similarity match between high-salient features of the base and low-salient features of the target. One line of evidence for salience imbalance in metaphor is the asymmetry in metaphoricity between a metaphorical comparison such as "Cigarettes are time bombs." and its reverse comparison "Time bombs are cigarettes." The effort to model similarity, whether literal (Tversky, 1977) or nonliteral, as in Ortony's work, as a partial match between componential representations is certainly compatible with the structure-mapping view. However, the representational assumption of ordered feature

lists does not easily allow for structural carryover. Feature-lists easily represent object attributes, but not relationships between objects.

Sternberg's (1977) theory of reasoning in simple $A:B::C:D$ analogies explicitly involves the mapping of objects and relations. Specifically, Sternberg proposes a sequence in which (1) attributes of A and B are encoded ("encoding"); (2) the relation between A and B is determined ("inference"); (3) C and D are encoded; (4) A is mapped onto C ("mapping") and C is mapped onto the possible solutions D, one attribute at a time, until a solution is reached in which the C-D relation is identical to the A-B relation. The basic assumptions about mapping are quite compatible with the approach offered here. (But see Grudin, 1980, for further discussion of the time sequence of processing.) However, to deal with complex analogies, with many objects and relations, a richer and more explicit theory of representation is needed.

In Sternberg's extension of his approach to metaphor, the mode of representation is that of multidimensional spaces. Like the Rumelhart and Abrahamson (1973) model of analogy, this theory is based on the notion of constructing parallel vectors in multidimensional spaces (Tourangeau & Sternberg, 1978). A metaphor such as "Therapists are priests." is understood by constructing an ideal vector from the origin

within the target subspace that is parallel to the original vector from origin to priest in the base subspace. The fit of the metaphor is then given by the distance between the ideal comparison concept found at the terminus of the vector and the actual target term. The closer the within-space fit and the greater the between-space distance, the more apt the metaphor will be. Thus, "Therapists are priests" is reasonably apt, because the between-space distance between medicine and religion is fairly large while the within-space dimensional positions of the particular base and target are quite close.

Like Ortony's salience imbalance theory, Sternberg's multidimensional space approach yields plausible predictions in the cases where the theory applies. However, since the spatial dimensions are basically adjectival (e.g., ferocity), it too is geared to comparisons of two objects taken as wholes. In both theories, all predicates are treated essentially as object-attributes, whether as dimensional distances or as entries on a list of features. Neither theory has any means of dealing with systems of interrelated objects or with objects decomposed into interrelated components. Causality, for example, is not representable. To capture complex metaphor and analogy, the representations of base and target must be made more explicit than either a feature-list representation or a multidimensional-space theory allow.

Miller's (1979) treatment is closest representationally to the present account. His propositional representations are essentially equivalent to those used here. Miller characterizes metaphors as comparison statements in which some aspects of the comparison are left unstated. His concern is the relation between the surface form of a metaphor and its underlying conceptual structure. For example, in a nominative metaphor, a term X is equated with Y when X is not a Y . The underlying structure postulated is $\text{EXIST}(F)$ and $\text{EXIST}(G)$ such that

$$(\text{sim}[F(x), G(y)]).$$

That is, there is some predicate F that applies to X that is similar to some predicate G that applies to Y .

Miller's approach differs in two main ways from a structure-mapping. First, Miller emphasizes similarity of predicates across domains, rather than identicalness of subpredicates. Second, no importance is attached to the distinction between higher-order and lower-order predicates: All predicates are equally likely to be mapped. In contrast, the distinction between attributes and relations, and between lower-order and higher-order relations, is central to the structure-mapping treatment. This difference stems partly from differences in content matter: Miller focuses on expressive metaphors, in which, as was argued earlier,

relatively more object-attributes are preserved than in explanatory analogies.

In summary, the ability to deal with analogies between complex systems is crucial to a characterization of scientific thinking. I have argued here that such systems analogies can be psychologically characterized as structure-mappings between propositionally represented domains. This framework allows us to state structural distinctions that distinguish good explanatory-predictive analogy from other kinds of metaphor. Applying these distinctions - clarity, richness, abstractness, base specificity and systematicity - allows a resolution of the unsatisfyingly simple dichotomy between the view that metaphoric thinking per se fosters scientific insight and the opposing view that metaphor necessarily leads to vague thinking. We can replace this dichotomy with a characterization of just what kind of metaphor is useful as explanatory-predictive analogy.

This framework is also useful in characterizing the structure of the naive mental models that people have about physical phenomena. These models appear to be more like expressive analogies than the analogies used by our expert subjects. This opens intriguing possibilities for further research: for example, do the models of experts differ from those of novices because of their greater experience with the

topic area, or is it the knowledge of general modelling rules that distinguishes experts from naive subjects, or both? We hope to pursue these questions in future research.

References

- Bobrow, D.G. Dimensions of representation. In D.G. Bobrow & A. Collins (Eds.), Representation and understanding: Studies in Cognitive Science. New York: Academic Press, 1975.
- Boyd, R. Metaphor and theory change: What is "metaphor" a metaphor for? In A. Ortony (Ed.), Metaphor and thought. Cambridge, England: Cambridge University Press, 1979, 356-408.
- Brown, J.S., & Burton, R.R. Multiple representations of knowledge for tutorial reasoning. In D.G. Bobrow & A. Collins (Eds.), Representation and understanding. New York: Academic Press, 1975.
- Brown, J.S., Collins, A., & Harris, G. Artificial intelligence and learning strategies. In H.F. O'Neil (Ed.), Learning strategies. New York: Academic Press, 1978.
- Cavendish, R. The black arts. New York: Capricorn Books, 1967.
- Collins, A.M., Stevens, A.L., & Brown, J.S. Multiple conceptual models of a complex system. Journal of Man-Machine Studies, 1979, in press.
- Collins, A.M., Stevens, A.L., & Goldin, S. Misconceptions in student's understanding. International Journal of Man-Machine Studies, 1979, 11, 145-156.
- deKleer, J. Multiple representations of knowledge in a mechanics problem solver. In the Proceedings of the Fifth International Joint Conference on Artificial Intelligence. Cambridge, Ma.: MIT, 1977, 299-304.
- Freud, S. On transformations of instinct as exemplified in anal eroticism. In J. Strachey (Ed.), The standard

edition of the complete psychological works of Sigmund Freud, Vol. XVII. London, The Hogarth Press Ltd. and the Institute of Psycho-Analysis, 1955. Reprinted in H.N. Mischel and W. Mischel (Eds.), Readings in Personality. New York: Holt, Rinehart & Winston, Inc., 1973, pp. 85-89.

Galileo. Dialogue concerning the two chief world systems - Ptolemaic and Copernican, published 1629; translated by S. Drake. Berkeley: University of California Press, 1967.

Gentner, D. Children's performance on a spatial analogies task. Child Development, 1977, 48, 1034-1039.

Gentner, D. If a tree had a knee, where would it be? Children's performance on simple spatial metaphors. Papers and Reports on Child Language Development, 1977, 13.

Gentner, D. & Collins, A.M. Mental models of physical systems. To appear in L. Talmy (Ed.), Conceptual frameworks: A cross-disciplinary perspective. In preparation.

Gick, M.L., & Holyoak, K.J. Analogical problem solving. Cognitive Psychology, in press.

Grudin, J. Processes in verbal analogy solution. Human Perception and Performance, 1980, in press.

Hesse, M.B. Models and analogies in science. Notre Dame, Indiana: University of Notre Dame Press, 1966.

Hoffman, R.R. Metaphor in science. In R.P. Honeck & R.R. Hoffman (Eds.), The psycholinguistics of figurative language. Hillsdale, New Jersey: Lawrence Erlbaum Associates, 1980.

Jaffe, B. Crucibles: The story of chemistry. New York: Dover Publications, 1976.

- Johnson-Laird, P.N. Mental models in cognitive science. Cognitive Science, 1980, 4, 71-115.
- Lakoff, G., & Johnson, M. Metaphors we live by, in press.
- Larkin, J.H. Problem solving in physics. Unpublished paper, Group in Science and Mathematics Education, Berkeley, California, 1977.
- Larkin, K.M. An analysis of adult procedure synthesis in fraction problems. Cambridge, Ma.: Bolt Beranek & Newman Inc, 1978.
- Miller, G.A. Images and models: Similes and metaphors. In A. Ortony (Ed.), Metaphor and thought. Cambridge, England: Cambridge University Press, 1979, 202-250.
- Ortony, A. The role of similarity in similes and metaphors. In A. Ortony (Ed.), Metaphor and thought. Cambridge, England: Cambridge University Press, 1979, 186-201.
- Palmer, S.E. Fundamental aspects of cognitive representation. In E. Rosch & B.B. Lloyd (Eds.), Cognition and categorization. Hillsdale, New Jersey: Erlbaum Associates, 1978, 259-303.
- Polya, G. Mathematics and plausible reasoning, Volume 1. Princeton, New Jersey: Princeton University Press, 1973.
- Pylyshyn, Z. Metaphorical imprecision and the "top-down" research strategy. In A. Ortony (Ed.), Metaphor and thought. Cambridge, England: Cambridge University Press, 1979, 420-436.
- Reif, F. Problem solving in physics or engineering: Human information processing and some teaching suggestions. American Society for Engineering Education, in press.

Rumelhart, D.E. Some problems with the notion of literal meaning. In A. Ortony (Ed.), Metaphor and thought. Cambridge, England: Cambridge University Press, 1979, 78-91.

Rumelhart, D.E., & Abrahamson, A.A. A model for analogical reasoning. Cognitive psychology, 1973, 5, 1-28.

Rumelhart, D.E., & Norman, D.A. The active structural network. In D.A. Norman, D.E. Rumelhart & the LNR Research Group, Explorations in Cognition. San Francisco: W.H. Freeman & Co., 1975.

Rumelhart, D.E., & Ortony, A. Representation of knowledge. In R. C. Anderson, R. J. Spiro, & W. E. Montague (Eds.), Schooling and the acquisition of knowledge. Hillsdale, N.J.: Lawrence Erlbaum Associates, 1977.

Schank, R., & Abelson, R. Scripts, plans, goals, and understanding. Hillsdale, N.J.: Lawrence Erlbaum Associates, 1977.

Schon, D.A. Generative metaphor: A perspective on problem-setting in social policy. In A. Ortony (Ed.), Metaphor and thought. Cambridge, England: Cambridge University Press, 1979, 254-283.

Searle, J.R. Metaphor. In A. Ortony (Ed.), Metaphor and thought. Cambridge, England: Cambridge University Press, 1979, 92-123.

Smith, B.C. Computational reflection. Doctoral dissertation, Electrical Engineering and Computer Science. M.I.T., in preparation.

Sternberg, R.J. Component processes in analogical reasoning. Psychological review, 1977, 84, 353-378.

Sternberg, R.J., Tourangeau, R., & Nigro, G. Metaphor, induction, and social policy: The convergence of

macroscopic and microscopic views. In A. Ortony (Ed.), Metaphor and thought. Cambridge, England: Cambridge University Press, 1979.

Tourangeau, R., & Sternberg, R.J. Understanding and appreciating metaphors. Technical Report No. 11, Department of Psychology, Yale University, June 1978.

Tversky, A. Features of similarity. Psychological Review, 1977, 84, 327-352.

Verbrugge, R.R., & McCarrell, N.S. Metaphoric comprehension: Studies in reminding and resembling. Cognitive psychology, 1977, 9, 494-533.

Footnotes

1. This research was supported the Department of the Navy, Office of Naval Research under Contract No. N00014-79-0338.

I would like to thank Allan Collins, Donald Gentner, Phil Kohn, Ed Smith, and Al Stevens, who collaborated on the development of these ideas, and Bernard Bahrs, Geoffrey Hinton, Yaakov Kareev, Donald Norman and David Rumelhart for their helpful comments on earlier versions of this paper. I also thank Brenda Starr for her help with all phases of the research and Cindy Hunt for preparing the manuscript.

2. An adequate discussion of literal similarity within this framework would require including a negative dependency on the number of nonshared features or predicates as well as the positive dependency on the number of shared predicates (Tversky, 1977). Tversky's valuable characterization of literal similarity does not utilize the relation-attribute distinction; all predicates are considered together, as "features". Whether the distinction is necessary for literal similarity remains to be seen.
3. Because of the hierarchical nature of schemata, there are two possible levels of object-attribution: that of the

component objects that make up the system (e.g., "The sun is round."; "Electrons are tiny.") or that of the system taken as a single holistic object (e.g., "The solar system is a huge whirling disc.").

4. One piece of groundwork is necessary before going on. The claim is that in interpreting analogies people make a significant distinction between predicates that express the relation between objects in the particular domain under consideration, and predicates that describe objects in and of themselves, independently of any particular system. To test this claim, one must derive the underlying predicate structure from a subject's surface language. In most cases, the form of the surface expression makes it clear whether the underlying predicate is an attribute or a relation. Predicates that take two or more objects, such as transitive verbs, express relationships between their arguments; adjectives often express single-object attributes. However, there are some subtleties that should be discussed. First, any stative dimension can be treated relationally, by making a comparison between objects instead of a single object description. For example, size is usually treated as an object attribute ("X is large." or "X is ten kilograms in mass."), but it can be used relationally in comparisons

("X is larger than Y." or "X is four kilograms greater in mass than Y."). There are clear surface signs of such relational uses - e.g., comparative inflections, presence of more than one noun argument - so they do not pose a serious classification problem.

One point that might seem to be problematic is that, as Palmer (1979) points out, object attribution statements often involve implicit comparisons with category norms. For example, "X is large." conveys a different size if X is a mouse than if X is an elephant. In effect, one is saying "X is large for a mouse." This might seem to undermine the distinction between attributes and relations. But regardless of what information enters into computing the meaning of an attribute-object pair, the key point here is how that meaning enters into an analogy interpretation: whether size, for example, is treated as an attribute or a relation within the domain of the analogy. Again, the predicate structure is inferred from the person's explicit statements. If, in interpreting the atom-solar system analogy, a subject wrote "The sun is large for a star.", then that relation would be considered part of the interpretation. In fact, no subject mentioned the size comparison between sun and other stars; they either mentioned the size relation between the sun and the

planets, or they mentioned the size of the sun as an object-attribute.

To a large extent, the underlying predicate structure of an analogic interpretation is inferable from explicit surface syntactic and inflectional patterns. However, a more difficult set of cases arises when underlying relations are expressed as surface attributes, through a process of abstraction (see Miller, 1979). For example, the adjective soporific, in "X is soporific." is stated as though it were a quality of X, but in fact conveys relational information: that there exist beings whom X puts to sleep. It stands for a set of relational statements like "X puts Y to sleep.", "X puts Z to sleep.". Such abstracted relational adjectives seem to convey both relational and attributional meaning.

5. It might be thought that, as with mathematical functions, one-to-many mappings would be disallowed but many-to-one mappings would be allowed. But this is not the case. Analogical clarity is violated by any many-to-one or one-to-many mapping in which the many are relationally distinct. As in the Rutherford model when any of the nine planets can map onto the same electron, it is fairly intuitive that a many-to-one mapping need pose no problem in clarity if the several objects in the base participate

in the same relationships. Similarly, a one-to-many mapping is not a problem if there is no structural reason to distinguish among the several target objects. For example, the solar system could reasonably be mapped onto helium, even though there is ambiguity as to which of helium's two electrons a given planet maps into. This ambiguity does not reduce clarity because the target objects -- the two electrons -- are, in terms of this analogy, relationally identical. A reduction in clarity occurs only when the choice makes a relational difference.

6. Mathematical models represent an extreme of clarity and abstractness as well as an extreme of base specificity. The set of mappable relations is strongly constrained, and the rules for concatenating relationships are well-specified. Once we choose a given mathematical system as base, we know thereby which combinatorial rules apply to relations in the base. This enormously simplifies the process of deriving new predictions to test in the target. We know, for example, that if the base relations are addition (R_1) and multiplication (R_2) over the integers, then we can expect distributivity to hold: $c(a+b) = ca + cb$, or

$$R_2[(c, R_1(a, b))] = R_1[R_2(c, a), R_2(c, b)]$$

A mathematical model tends to predict a small number of relational types, which are well-specified and systematic enough to be concatenated into long chains of prediction.

7. I hope it is understood that I am not accusing Shakespeare or Eliot of simple-mindedness; indeed in this particular case the shifting metaphors may help convey Romeo's agitation. The point is simply that expressive analogies can fulfil their function without being clear and systematic, whereas explanatory analogies cannot.
8. A Freudian defense might claim that the theory merely reflects the illogic of the unconscious. But a theory about an unclarified thought process need not itself be unclarified.

Navy

- 1 Meryl S. Baker
NPRDC
Code P309
San Diego, CA 92152
- 1 Dr. Robert Breaux
Code N-711
NAVTRAEQUIPCEN
Orlando, FL 32813
- 1 Chief of Naval Education and Training
Liason Office
Air Force Human Resource Laboratory
Flying Training Division
WILLIAMS AFB, AZ 85224
- 1 DR. PAT FEDERICO
NAVY PERSONNEL R&D CENTER
SAN DIEGO, CA 92152
- 1 Dr. John Ford
Navy Personnel R&D Center
San Diego, CA 92152
- 1 LT Steven D. Harris, MSC, USN
Code 6021
Naval Air Development Center
Warminster, Pennsylvania 18974
- 1 Dr. Patrick R. Harrison
Psychology Course Director
LEADERSHIP & LAW DEPT. (7b)
DIV. OF PROFESSIONAL DEVELOPMENT
U.S. NAVAL ACADEMY
ANNAPOLIS, MD 21402
- 1 Dr. Norman J. Kerr
Chief of Naval Technical Training
Naval Air Station Memphis (75)
Millington, TN 38054
- 1 Dr. William L. Maloy
Principal Civilian Advisor for
Education and Training
Naval Training Command, Code 00A
Pensacola, FL 32508

Navy

- 1 Dr. Kneale Marshall
Scientific Advisor to DCNO(MPT)
OP01T
Washington DC 20370
- 1 CAPT Richard L. Martin, USN
Prospective Commanding Officer
USS Carl Vinson (CVN-70)
Newport News Shipbuilding and Drydock Co
Newport News, VA 23607
- 1 Dr William Montague
Navy Personnel R&D Center
San Diego, CA 92152
- 1 Commanding Officer
U.S. Naval Amphibious School
Coronado, CA 92155
- 1 Library
Naval Health Research Center
P. O. Box 85122
San Diego, CA 92138
- 1 Naval Medical R&D Command
Code 44
National Naval Medical Center
Bethesda, MD 20014
- 1 CAPT Paul Nelson, USN
Chief, Medical Service Corps
Bureau of Medicine & Surgery (MED-23)
U. S. Department of the Navy
Washington, DC 20372
- 1 Ted M. I. Yellen
Technical Information Office, Code 201
NAVY PERSONNEL R&D CENTER
SAN DIEGO, CA 92152
- 1 Library, Code P201L
Navy Personnel R&D Center
San Diego, CA 92152
- 6 Commanding Officer
Naval Research Laboratory
Code 2627
Washington, DC 20390

Navy

- 1 Psychologist
ONR Branch Office
Bldg 114, Section D
666 Summer Street
Boston, MA 02210
- 1 Psychologist
ONR Branch Office
536 S. Clark Street
Chicago, IL 60605
- 1 Office of Naval Research
Code 437
800 N. Quincy SStreet
Arlington, VA 22217
- 5 Personnel & Training Research Programs
(Code 458)
Office of Naval Research
Arlington, VA 22217
- 1 Psychologist
ONR Branch Office
1030 East Green Street
Pasadena, CA 91101
- 1 Office of the Chief of Naval Operations
Research, Development, and Studies Branc 1
(OP-102)
Washington, DC 20350
- 1 Captain Donald F. Parker, USN
Commanding Officer
Navy Personnel R&D Center
San Diego, CA 92152
- 1 LT Frank C. Petho, MSC, USN (Ph.D)
Code L51
Naval Aerospace Medical Research Laborat
Pensacola, FL 32508
- 1 DR. RICHARD A. POLLAK
ACADEMIC COMPUTING CENTER
U.S. NAVAL ACADEMY
ANNAPOLIS, MD 21402

Navy

- 1 Mr. Arnold Rubenstein
Naval Personnel Support Technology
Naval Material Command (08T244)
Room 1044, Crystal Plaza #5
2221 Jefferson Davis Highway
Arlington, VA 20360
- 1 Dr. Worth Scanland
Chief of Naval Education and Training
Code N-5
NAS, Pensacola, FL 32508
- 1 Dr. Robert G. Smith
Office of Chief of Naval Operations
OP-987H
Washington, DC 20350
- 1 Dr. Alfred F. Smode
Training Analysis & Evaluation Group
(TAEG)
Dept. of the Navy
Orlando, FL 32813
- 1 Dr. Richard Sorensen
Navy Personnel R&D Center
San Diego, CA 92152
- 1 Dr. Robert Wisher
Code 309
Navy Personnel R&D Center
San Diego, CA 92152
- 1 Mr John H. Wolfe
Code P310
U. S. Navy Personnel Research and
Development Center
San Diego, CA 92152

Army

- 1 Technical Director
U. S. Army Research Institute for the
Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333
- 1 HQ USAREUE & 7th Army
ODCSOPS
USAAREUE Director of GED
APO New York 09403
- 1 DR. RALPH DUSEK
U.S. ARMY RESEARCH INSTITUTE
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333
- 1 Col Frank Hart
Army Research Institute for the
Behavioral & Social Sciences
5001 Eisenhower Blvd.
Alexandria, VA 22333
- 1 Dr. Michael Kaplan
U.S. ARMY RESEARCH INSTITUTE
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333
- 1 Dr. Milton S. Katz
Training Technical Area
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333
- 1 Dr. Harold F. O'Neil, Jr.
Attn: PERI-OK
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333
- 1 LTC Michael Plummer
Chief, Leadership & Organizational
Effectiveness Division
Office of the Deputy Chief of Staff
for Personnel
Dept. of the Army
Pentagon, Washington DC 20301

Army

- 1 Dr. Robert Sasmor
U. S. Army Research Institute for the
Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333
- 1 Commandant
US Army Institute of Administration
Attn: Dr. Sherrill
FT Benjamin Harrison, IN 46256
- 1 Dr. Joseph Ward
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Air Force

- 1 Air University Library
AUL/LSE 76/443
Maxwell AFB, AL 36112
- 1 Dr. Earl A. Alluisi
HQ, AFHRL (AFSC)
Brooks AFB, TX 78235
- 1 DR. T. E. COTTERMAN
AFHRL/ASR
WRIGHT PATTERSON AFB
OHIO 45433
- 1 Dr. Genevieve Haddad
Program Manager
Life Sciences Directorate
AFOSR
Bolling AFB, DC 20332
- 1 Dr. Ross L. Morgan (AFHRL/LR)
Wright -Patterson AFB
Ohio 45433
- 1 Dr. Marty Rockway (AFHRL/TT)
Lowry AFB
Colorado 80230
- 1 Dr. Frank Schufletowski
U.S. Air Force
ATC/XPTD
Randolph AFB, TX 78148
- 2 3700 TCHTW/TTGH Stop 32
Sheppard AFB, TX 76311
- 1 Jack A. Thorpe, Maj., USAF
Naval War College
Providence, RI 02846

Marines

- 1 H. William Greenup
Education Advisor (E031)
Education Center, MCDEC
Quantico, VA 22134
- 1 Special Assistant for Marine
Corps Matters
Code 100M
Office of Naval Research
800 N. Quincy St.
Arlington, VA 22217
- 1 DR. A.L. SLAFKOSKY
SCIENTIFIC ADVISOR (CODE RD-1)
HQ, U.S. MARINE CORPS
WASHINGTON, DC 20380

CoastGuard

- 1 Chief, Psychological Research Branch
U. S. Coast Guard (G-P-1/2/TP42)
Washington, DC 20593

Other DoD

- 12 Defense Documentation Center
Cameron Station, Bldg. 5
Alexandria, VA 22314
Attn: TC
- 1 Dr. Craig I. Fields
Advanced Research Projects Agency
1400 Wilson Blvd.
Arlington, VA 22209
- 1 Dr. Dexter Fletcher
ADVANCED RESEARCH PROJECTS AGENCY
1400 WILSON BLVD.
ARLINGTON, VA 22209
- 1 Military Assistant for Training and
Personnel Technology
Office of the Under Secretary of Defense
for Research & Engineering
Room 3D129, The Pentagon
Washington, DC 20301
- 1 HEAD, SECTION ON MEDICAL EDUCATION
UNIFORMED SERVICES UNIV. OF THE
HEALTH SCIENCES
6917 ARLINGTON ROAD
BETHESDA, MD 20014

Civil Govt

- 1 Dr. Susan Chipman
Learning and Development
National Institute of Education
1200 19th Street NW
Washington, DC 20208
- 1 Mr. James M. Ferstl
Bureau of Training
U.S. Civil Service Commission
Washington, D.C. 20415
- 1 Dr. Joseph I. Lipson
SEDR W-638
National Science Foundation
Washington, DC 20550
- 1 Dr. John Mays
National Institute of Education
1200 19th Street NW
Washington, DC 20208
- 1 William J. McLaurin
Rm. 301, Internal Revenue Service
2221 Jefferson Davis Highway
Arlington, VA 22202
- 1 Dr. Arthur Melmed
National Institute of Education
1200 19th Street NW
Washington, DC 20208
- 1 Dr. Andrew R. Molnar
Science Education Dev.
and Research
National Science Foundation
Washington, DC 20550
- 1 Dr. H. Wallace Sinaiko
Program Director
Manpower Research and Advisory Services
Smithsonian Institution
801 North Pitt Street
Alexandria, VA 22314
- 1 Dr. Frank Withrow
U. S. Office of Education
400 Maryland Ave. SW
Washington, DC 20202

Civil Govt

- 1 Dr. Joseph L. Young, Director
Memory & Cognitive Processes
National Science Foundation
Washington, DC 20550

Non Govt

- 1 Dr. John R. Anderson
Department of Psychology
Carnegie Mellon University
Pittsburgh, PA 15213
- 1 Dr. John Annett
Department of Psychology
University of Warwick
Coventry CV4 7AL
ENGLAND
- 1 DR. MICHAEL ATWOOD
SCIENCE APPLICATIONS INSTITUTE
40 DENVER TECH. CENTER WEST
7935 E. PRENTICE AVENUE
ENGLEWOOD, CO 80110
- 1 1 psychological research unit
Dept. of Defense (Army Office)
Campbell Park Offices
Canberra ACT 2600, Australia
- 1 Dr. R. A. Avner
University of Illinois
Computer-Based Educational Research Lab
Urbana, IL 61801
- 1 Dr. Alan Baddeley
Medical Research Council
Applied Psychology Unit
15 Chaucer Road
Cambridge CB2 2EF
ENGLAND
- 1 Dr. Patricia Baggett
Department of Psychology
University of Denver
University Park
Denver, CO 80208
- 1 Ms. Carole A. Bagley
Minnesota Educational Computing
Consortium
2354 Hidden Valley Lane
Stillwater, MN 55082

Non Govt

- 1 Mr Avron Barr
Department of Computer Science
Stanford University
Stanford, CA 94305
- 1 Dr. John Bergan
School of Education
University of Arizona
Tuscon AZ 85721
- 1 Dr. Nicholas A. Bond
Dept. of Psychology
Sacramento State College
600 Jay Street
Sacramento, CA 95819
- 1 Dr. Lyle Bourne
Department of Psychology
University of Colorado
Boulder, CO 80309
- 1 Dr. Kenneth Bowles
Institute for Information Sciences
University of California at San Diego
La Jolla, CA 92037
- 1 Dr. John S. Brown
XEROX Palo Alto Research Center
3333 Coyote Road
Palo Alto, CA 94304
- 1 Dr. Bruce Buchanan
Department of Computer Science
Stanford University
Stanford, CA 94305
- 1 DR. C. VICTOR BUNDERSON
WICAT INC.
UNIVERSITY PLAZA, SUITE 10
1160 SO. STATE ST.
OREM, UT 84057
- 1 Dr. Anthony Cancelli
School of Education
University of Arizona
Tuscon, AZ 85721

Non Govt

- 1 Dr. Pat Carpenter
Department of Psychology
Carnegie-Mellon University
Pittsburgh, PA 15213
- 1 Dr. John B. Carroll
Psychometric Lab
Univ. of No. Carolina
Davie Hall 013A
Chapel Hill, NC 27514
- 1 Charles Myers Library
Livingstone House
Livingstone Road
Stratford
London E15 2LJ
ENGLAND
- 1 Dr. William Chase
Department of Psychology
Carnegie Mellon University
Pittsburgh, PA 15213
- 1 Dr. Micheline Chi
Learning R & D Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15213
- 1 Dr. William Clancey
Department of Computer Science
Stanford University
Stanford, CA 94305
- 1 Dr. Lynn A. Cooper
Department of psychology
Uris Hall
Cornell University
Ithaca, NY 14850
- 1 Dr. Meredith P. Crawford
American Psychological Association
1200 17th Street, N.W.
Washington, DC 20036
- 1 Dr. Kenneth B. Cross
Anacapa Sciences, Inc.
P.O. Drawer Q
Santa Barbara, CA 93102

Non Govt

- 1 Dr. Hubert Dreyfus
Department of Philosophy
University of California
Berkeley, CA 94720
- 1 LCOL J. C. Eggenberger
DIRECTORATE OF PERSONNEL APPLIED RESEARCH
NATIONAL DEFENCE HQ
101 COLONEL BY DRIVE
OTTAWA, CANADA K1A 0K2
- 1 ERIC Facility-Acquisitions
4833 Rugby Avenue
Bethesda, MD 20014
- 1 Dr. A. J. Eschenbrenner
Dept. E422, Bldg. 81
McDonnell Douglas Astronautics Co.
P.O.Box 516
St. Louis, MO 63166
- 1 Dr. Ed Feigenbaum
Department of Computer Science
Stanford University
Stanford, CA 94305
- 1 Mr. Wallace Feurzeig
Bolt Beranek & Newman, Inc.
50 Moulton St.
Cambridge, MA 02138
- 1 Dr. Victor Fields
Dept. of Psychology
Montgomery College
Rockville, MD 20850
- 1 DR. JOHN D. FOLLEY JR.
APPLIED SCIENCES ASSOCIATES INC
VALENCIA, PA 16059
- 1 Dr. John R. Frederiksen
Bolt Beranek & Newman
50 Moulton Street
Cambridge, MA 02138

Non Govt

- 1 Dr. Alinda Friedman
Department of Psychology
University of Alberta
Edmonton, Alberta
CANADA T6G 2E9
- 1 Dr. R. Edward Geiselman
Department of Psychology
University of California
Los Angeles, CA 90024
- 1 DR. ROBERT GLASER
LRDC
UNIVERSITY OF PITTSBURGH
3939 O'HARA STREET
PITTSBURGH, PA 15213
- 1 Dr. Marvin D. Glock
217 Stone Hall
Cornell University
Ithaca, NY 14853
- 1 Dr. Daniel Gopher
Industrial & Management Engineering
Technion-Israel Institute of Technology
Haifa
ISRAEL
- 1 DR. JAMES G. GREENO
LRDC
UNIVERSITY OF PITTSBURGH
3939 O'HARA STREET
PITTSBURGH, PA 15213
- 1 Dr. Harold Hawkins
Department of Psychology
University of Oregon
Eugene OR 97403
- 1 Dr. Barbara Hayes-Roth
The Rand Corporation
1700 Main Street
Santa Monica, CA 90406
- 1 Dr. Frederick Hayes-Roth
The Rand Corporation
1700 Main Street
Santa Monica, CA 90406

Non Govt

- 1 Dr. Dustin H. Heuston
Wicat, Inc.
Box 986
Orem, UT 84057
- 1 Dr. James R. Hoffman
Department of Psychology
University of Delaware
Newark, DE 19711
- 1 Glenda Greenwald, Ed.
"Human Intelligence Newsletter"
P. O. Box 1163
Birmingham, MI 48012
- 1 Library
HumRRO/Western Division
27857 Berwick Drive
Carmel, CA 93921
- 1 Dr. Earl Hunt
Dept. of Psychology
University of Washington
Seattle, WA 98105
- 1 DR. KAY INABA
21116 VANOWEN ST
CANOGA PARK, CA 91303
- 3 Journal Supplement Abstract Service
American Psychological Association
1200 17th Street N.W.
Washington, DC 20036
- 1 Dr. Wilson A. Judd
McDonnell-Douglas
Astronautics Co.-St. Louis
P.O. Box 30204
Lowry AFB, CO 80230
- 1 Dr. Steven W. Keele
Dept. of Psychology
University of Oregon
Eugene, OR 97403
- 1 Dr. Walter Kintsch
Department of Psychology
University of Colorado
Boulder, CO 80302

Non Govt

- 1 Dr. David Kieras
Department of Psychology
University of Arizona
Tuscon, AZ 85721
- 1 Dr. Kenneth A. Klivington
Program Officer
Alfred P. Sloan Foundation
630 Fifth Avenue
New York, NY 10111
- 1 Dr. Mazie Knerr
Litton-Mellonics
Box 1286
Springfield, VA 22151
- 1 Dr. Stephen Kosslyn
Harvard University
Department of Psychology
33 Kirkland Street
Cambridge, MA 02138
- 1 Mr. Marlin Kroger
1117 Via Goleta
Palos Verdes Estates, CA 90274
- 1 Dr. Jill Larkin
Department of Psychology
Carnegie Mellon University
Pittsburgh, PA 15213
- 1 Dr. Alan Lesgold
Learning R&D Center
University of Pittsburgh
Pittsburgh, PA 15260
- 1 Dr. Michael Levine
210 Education Building
University of Illinois
Champaign, IL 61820
- 1 Dr. Charles Lewis
Faculteit Sociale Wetenschappen
Rijksuniversiteit Groningen
Oude Boteringestraat
Groningen
NETHERLANDS

Non Govt

- 1 Dr. Mark Miller
Computer Science Laboratory
Texas Instruments, Inc.
Mail Station 371, P.O. Box 225936
Dallas, TX 75265
- 1 Dr. Allen Munro
Behavioral Technology Laboratories
1845 Elena Ave., Fourth Floor
Redondo Beach, CA 90277
- 1 Dr. Donald A Norman
Dept. of Psychology C-009
Univ. of California, San Diego
La Jolla, CA 92093
- 1 Dr. Jesse Orlansky
Institute for Defense Analyses
400 Army Navy Drive
Arlington, VA 22202
- 1 Dr. Seymour A. Papert
Massachusetts Institute of Technology
Artificial Intelligence Lab
545 Technology Square
Cambridge, MA 02139
- 1 Dr. James A. Paulson
Portland State University
P.O. Box 751
Portland, OR 97207
- 1 MR. LUIGI PETRULLO
2431 N. EDGEWOOD STREET
ARLINGTON, VA 22207
- 1 DR. PETER POLSON
DEPT. OF PSYCHOLOGY
UNIVERSITY OF COLORADO
BOULDER, CO 80309
- 1 DR. DIANE M. RAMSEY-KLEE
R-K RESEARCH & SYSTEM DESIGN
3947 RIDGEMONT DRIVE
MALIBU, CA 90265

Non Govt

- 1 Dr. Fred Reif
SESAME
c/o Physics Department
University of California
Berkeley, CA 94720
- 1 Dr. Andrew M. Rose
American Institutes for Research
1055 Thomas Jefferson St. NW
Washington, DC 20007
- 1 Dr. Ernst Z. Rothkopf
Bell Laboratories
600 Mountain Avenue
Murray Hill, NJ 07974
- 1 Dr. David Rumelhart
Center for Human Information Processing
Univ. of California, San Diego
La Jolla, CA 92093
- 1 DR. WALTER SCHNEIDER
DEPT. OF PSYCHOLOGY
UNIVERSITY OF ILLINOIS
CHAMPAIGN, IL 61820
- 1 Dr. Alan Schoenfeld
Department of Mathematics
Hamilton College
Clinton, NY 13323
- 1 DR. ROBERT J. SEIDEL
INSTRUCTIONAL TECHNOLOGY GROUP
HUMRRO
300 N. WASHINGTON ST.
ALEXANDRIA, VA 22314
- 1 Committee on Cognitive Research
% Dr. Lonnie R. Sherrod
Social Science Research Council
605 Third Avenue
New York, NY 10016
- 1 Robert S. Siegler
Associate Professor
Carnegie-Mellon University
Department of Psychology
Schenley Park
Pittsburgh, PA 15213

Non Govt

- 1 Dr. Robert Smith
Department of Computer Science
Rutgers University
New Brunswick, NJ 08903
- 1 Dr. Richard Snow
School of Education
Stanford University
Stanford, CA 94305
- 1 Dr. Kathryn T. Spoehr
Department of Psychology
Brown University
Providence, RI 02912
- 1 Dr. Robert Sternberg
Dept. of Psychology
Yale University
Box 11A, Yale Station
New Haven, CT 06520
- 1 DR. ALBERT STEVENS
BOLT BERANEK & NEWMAN, INC.
50 MOULTON STREET
CAMBRIDGE, MA 02138
- 1 Mr. William Stobie
McDonnell-Douglas
Astronautics Co.
P. O. Box 30204
Chico, CA 95926
- 1 Dr. David Stone
ED 236
SUNY, Albany
Albany, NY 12222
- 1 DR. PATRICK SUPPES
INSTITUTE FOR MATHEMATICAL STUDIES IN
THE SOCIAL SCIENCES
STANFORD UNIVERSITY
STANFORD, CA 94305
- 1 Dr. Kikumi Tatsuoka
Computer Based Education Research
Laboratory
252 Engineering Research Laboratory
University of Illinois
Urbana, IL 61801

Non Govt

- 1 Dr. John Thomas
IBM Thomas J. Watson Research Center
P.O. Box 218
Yorktown Heights, NY 10598
- 1 DR. PERRY THORNDYKE
THE RAND CORPORATION
1700 MAIN STREET
SANTA MONICA, CA 90406
- 1 Dr. Walt W. Tornow
Control Data Corporation
Corporate Personnel Research
P.O. Box 0 - HQN060
Minneapolis, MN 55440
- 1 Dr. Douglas Towne
Univ. of So. California
Behavioral Technology Labs
1845 S. Elena Ave.
Redondo Beach, CA 90277
- 1 Dr. J. Uhlener
Perceptronics, Inc.
6271 Variel Avenue
Woodland Hills, CA 91364
- 1 Dr. Phyllis Weaver
Graduate School of Education
Harvard University
10 Larsen Hall, Appian Way
Cambridge, MA 02138
- 1 Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455
- 1 DR. GERSHON WELTMAN
PERCEPTRONICS INC.
6271 VARIEL AVE.
WOODLAND HILLS, CA 91367
- 1 DR. SUSAN E. WHITELEY
PSYCHOLOGY DEPARTMENT
UNIVERSITY OF KANSAS
LAWRENCE, KANSAS 66044

Non Govt

- 1 Dr. Karl Zinn
Center for research on Learning
and Teaching
University of Michigan
Ann Arbor, MI 48104

ATE
LMED
-8